A Model for Evaluation of Peak Pricing of Transport Facilities

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ABSTRACT

When a charge is imposed on users of a transport facility during the peak period, users must either travel in the peak and pay the charge, travel off peak and avoid the charge, or not travel at all. This paper describes a model for predicting users reactions to a peak charge, and the costs associated with these reactions. The model is based on the concepts of trip surplus and time diversion cost. While the model has not been validated, it gives plausible results under a range of conditions.

INTRODUCTION

Peak Pricing

The demand for many transport facilities exhibits significant fluctuation in time. For roads, railways and airports, there is generally a fluctuation over the hours of the day, the days of the week, and, to some extent, the seasons of the year.

As the capacity required for each facility is determined by the peak demand, rather than some sort of average demand, it is economically desirable to avoid highly 'peaked' demand profiles.

In situations where the natural demand is highly peaked, economists have long argued that the most efficient solution is to use some form of peak pricing; i.e., to impose a surcharge to take account of higher marginal costs of peak operations, so as to encourage users to travel during off peak times. This leads to the desirable situation in which those users who place a high value on travelling during the peak hours do so, and those for whom it is not so important travel in off peak times.

When peak pricing is applied to any transport facility, each peak user can react in one of three ways:

i) continue to travel during the peak and pay the peak charge (STAY)

ii) travel during off-peak time (DIVERT)

iii) not travel at all (SUPPRESS).

To evaluate any proposed peak pricing scheme, it is necessary to establish:

i) the proportions of peak users who fall in each of the above three categories

ii) the economic costs associated with each.

The Model

The purpose of this paper is to describe a model to carry out the above evaluation. This model is based on the concepts of:

i) trip surplus, derived from the travel demand curve; and

ii) time diversion cost, which is the cost to the user of having to travel at other than his preferred time.

It assumes that users make their decisions on whether to stay, divert, or suppress on a rational economic basis, in terms of their trip surplus and time diversion cost.

The model has been used under a range of conditions and gives plausible results for user reactions. However, attempts at validation were limited, as there is very little real data available.

Thus the model must at this stage be treated as unvalidated. However, there is very little in the literature on this question, and virtually no reports of applications of peak pricing. This paper is therefore written to provide:

i) a framework for possible validation at a later stage

ii) some order of magnitude numbers, to be used with caution, in situations where there are decisions to be made and no data is available.

Outline of Paper

The paper is in five main sections. Following this introduction, the second section gives a brief review of the literature relevant to peak pricing. The third section describes the model and the assumptions used in it. The fourth section gives tabulations of results obtained from the model under a range of conditions, and the final section discusses the validity of the model. An appendix gives the mathematical development of the model.

PREVIOUS WORK

This section gives a brief outline of some of the published work on peak pricing.

Most of the early work was carried out for application to the electric power Industry. A welfare economic basis was introduced by Williamson (1966), in his analysis of peak pricing under indivisibility constraints.

Park (1971) examined the use of tolls for congested airports, but considered only a single period (the peak) in his analysis. He assumed that the demand curve for travel in this period was known, and ignored the off-peak period.

Jackson (1971) considered the problems of noise and congestion at airports, using a two period (peak and off-peak) day, and determined the charge for each period to maximise consumer surplus, subject to limits on congestion and noise. He assumed, however, that the two periods were independent, and that each had its own price elasticity.

Forsyth (1972), in his discussion of the timing of the investment in the third London Airport, claimed that pricing is a significant factor, and that a peak pricing scheme could affect this timing. He recognised the need to obtain cross elasticities between periods, but said that no data was available.

Finally, Little and McLeod (1972) discuss the British Airports Authority's new pricing policy which includes a peak charge. There is, however, no data available on the results of this policy.

It thus appears that there is little theoretical or practical work on the time elasticity of demand; i.e. the extent to which a peak charge will induce users to travel during off peak times.

This paper provides an initial step in this direction.

DESCRIPTION OF MODEL

Basic Model of User Behaviour

While the model is general, it is convenient to think of it as applied to a single trip, for which the cost F to the user consists of the direct charge (fare) plus the value of the user's time for the trip. In addition, there is a peak period surcharge T for users who travel in the peak.

Each user whose preferred travel time is in the peak can be described by :

i) a gross value V which he places on the trip (made at his preferred time)

ii) a time diversion cost D, which represents the cost, or loss of value, to the user from having to travel at an off peak time rather than at his preferred time.

The quantity V-F is known as the trip surplus, and represents the net gain to the user from making the trip. Then the assumption of rational economic behaviour implies that:

i) the user will make the trip in the peak (STAY) if
   a) his time diversion cost is greater than the peak charge, i.e. $D > T$; and
   b) the trip is still worthwhile if he has to pay the peak charge; i.e. $S > T$

ii) the user will make the trip in the off peak time (DIVERT) if
   a) his time diversion cost is less than the peak charge, i.e. $D < T$; and
   b) the trip is still worthwhile; i.e. $S > D$

iii) the user will not travel at all (SUPPRESS) if his surplus is less than both the peak charge and his time diversion cost; i.e. $S < T$ and $S < D$

Whether a particular user stays, diverts, or suppresses depends, therefore, on his surplus S and time diversion cost D. To establish the overall behaviour of a group of users, it is necessary to establish the distribution of S and D in a typical group.

User Distributions

Distribution of Surplus It is assumed that the demand for the trip concerned has constant elasticity r; i.e., that the demand curve is of the form:

$$Q = kP^{-r}$$

where Q is the number of users who wish to make the trip when the generalised cost (fare + time) is P, and k is some constant. Alternatively, Q may be regarded as the number of users whose trip value V is greater than P. This demand curve is illustrated in Fig. 1.

From this assumed demand curve, it is possible to derive the distribution of trip surplus g(S)[1] of the N users who travel when the generalised trip cost P is equal to F. This distribution is:

$$g(S) = \frac{r\,F^{r}}{(F + S)^{r+1}} \qquad 0 \leqslant S \leqslant \infty$$

and it is derived by the following argument.

From Fig. 1, it can be seen that the number of users with surplus between S and S + dS is dQ, where

$$dQ = -\frac{dQ}{dP}\,dS \qquad (dS = dP)$$

1. g(S) is defined to be such that the proportion of users with surplus between S and S + dS is g(S) dS.

Thus the <u>proportion</u> of users, $\frac{dQ}{N}$, is

$$-\frac{dQ}{NdP} \quad dS$$

Thus

$$g(S) = -\frac{1}{N} \frac{dQ}{dP}$$

$$= -\frac{1}{kF^{-r}} \quad (-rkP^{-r+1})$$

$$= \frac{rF^{r}}{(F+S)^{r+1}}$$

This distribution implies that the proportion of the population with a given level of surplus decreases as the level of surplus increases.

<u>Distribution of Time Diversion Cost h(D)</u> For some passengers, the time diversion cost is likely to be negligible, while for others it may be substantial. It was assumed that the time diversion cost is uniformly distributed: i.e.,

$$h(D) = \frac{1}{C} \quad 0 \leq D \leq C$$

where

C = n H U

n = range parameter

H = hours of diversion

U = passenger time value ($/hour)

There was no firm basis for the selection of this particular form for the distribution. It seemed reasonable, and gave plausible results. It could be that different forms are appropriate for different applications. The selection of the value of n, the range parameter, is discussed in the final section.

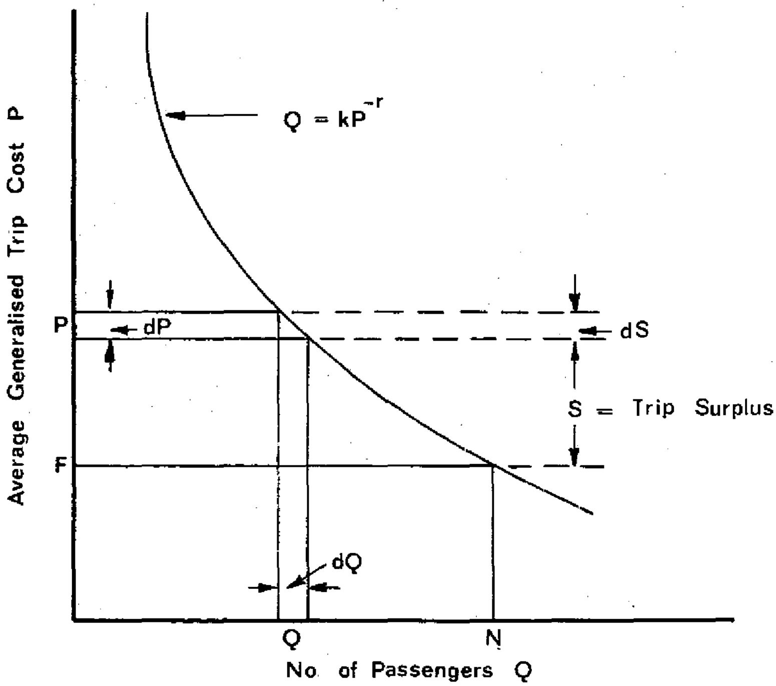

FIGURE 1

TRIP DEMAND CURVE AND

DERIVATION OF TRIP SURPLUS DISTRIBUTION

<u>Joint Distribution of S and D</u> It was assumed that the surplus and the time diversion cost were independent, so that the joint distribution was

$$f(S,D) \quad = \quad \frac{r\ F^{r}}{C\ (F + S)^{r + 1}} \qquad \begin{array}{l} 0 \leqslant S \leqslant \infty \\ 0 \leqslant D \leqslant C \end{array}$$

This assumption of independence was made in the absence of data, and is discussed in the final section.

<u>Response to Peak Charge</u>

From the above joint distribution, and the model of individual behaviour, it is possible to establish the overall response to a peak charge; i.e., to establish the proportions of users who stay, divert and suppress, and the costs associated with each. This is done by integrating appropriate functions of S and D.

These integrations are straightforward, but lengthy, and so are carried out in the Appendix. Only the final results are given here.

<u>Notation</u> It is convenient to define a number of quantities which simplify the expressions. The quantities used are:

F = generalised cost of trip

T = peak charge

C = maximum time diversion cost

r = elasticity of demand

a = T/F

b = T/C

$$L_1 = \begin{cases} \dfrac{1 - (1 + a)^{1 - r}}{r - 1} & r \neq 1 \\ \ln(1 + a) & r = 1 \end{cases}$$

$$L_2 = \begin{cases} \dfrac{1 - (1 + a)^{2 - r}}{r - 2} & r \neq 2 \\ \ln(1 + a) & r = 2 \end{cases}$$

Proportions The proportions of users who stay, divert, and suppress are found by integrating the distribution f(S,D) over the appropriate regions in the S - D plane, as shown in Fig. 2.

Users who stay are those for which D > T and S > T. The proportion is thus

$$\text{STAY} = \int_{D=T}^{C} \int_{S=T}^{\infty} f(S, D)\, dS\, dD \qquad (1)$$

$$= \frac{1 - b}{(1+a)^r}$$

Users who divert are those for which D > T and S > D. The proportion is:

$$\text{DIVERT} = \int_{D=0}^{T} \int_{S=D}^{\infty} f(S, D)\, dS\, dD \qquad (2)$$

$$= \frac{b}{a} L_1$$

Users who suppress are those for which S < T and S < D. The proportion is:

$$\text{SUPPRESS} = \int_{S=0}^{T} \int_{D=S}^{T} f(S, D)\, dD\, dS \qquad (3)$$

$$= 1 - \frac{1 - b}{(1+a)^r} - \frac{b}{a} L_1$$

It may be noted that these three proportions necessarily sum to 1.

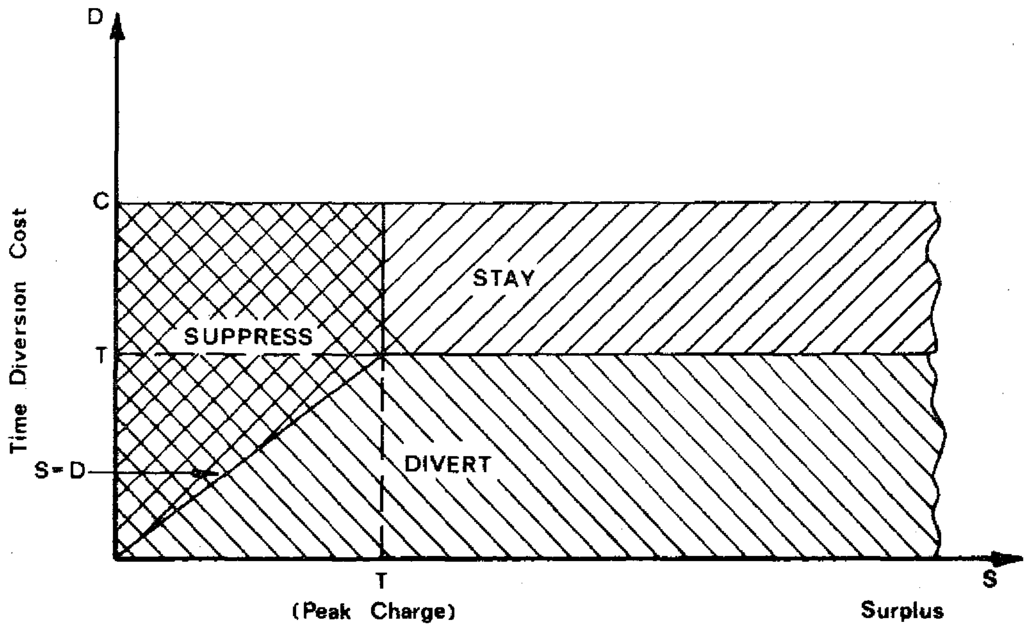

FIGURE 2

USER DECISIONS IN TERMS OF SURPLUS AND TIME DIVERSION COSTS

Costs Resulting From Peak Charge

The average cost per user in each of the three groups incurred as a result of the peak charge is calculated by dividing the total cost for the group by the proportion of users in the group. These costs are most conveniently expressed as a fraction of the trip cost F.

Cost to Users who Stay Those users who travel in the peak experience a cost equal to the peak charge T. Thus:

$$\text{COST/STAYER} = T = aF$$

Cost to Users who Divert Those users who elect to travel off peak incur a cost equal to their time diversion cost. The total cost for these users is:

$$\text{TOTAL DIVERTER COST} = \int_{D=0}^{T} D \int_{S=D}^{\infty} f(S, D)\, dS\, dD \quad (4)$$

$$= \frac{b}{a}(L_2 - L_1)\, F$$

Thus, from the proportion of diverters,

$$\text{COST/DIVERTER} = \frac{L_2 - L_1}{L_1}\, F$$

Cost to Users who Suppress Those who suppress incur a cost equal to their trip surplus. The total cost for these users is:

$$\text{TOTAL SUPPRESSER COST} = \int_{S=0}^{T} S \int_{D=S}^{T} f(S, D)\, dD\, dS \quad (5)$$

$$= \left\{ L_1 - \frac{a(1-b)}{(1+a)^r} - \frac{2b}{a}(L_2 - L_1) \right\} F$$

Thus, from the proportion of suppressers:

COST/SUPPRESSER

$$= \frac{L_1 - \frac{a(1-b)}{(1+a)^r} - \frac{2b}{a}(L_2 - L_1)}{1 - \frac{1-b}{(1+a)^r} - \frac{b}{a} L_1} F$$

Average Cost Per user The total cost to users, averaged over all users, is the sum of the above total costs; i.e.

AVERAGE USER COST

$$= \frac{a(1-b)}{(1+a)^r} F + \frac{b}{a}(L_2 - L_1) F$$

$$+ \left\{ L_1 - \frac{a(1-b)}{(1+a)^r} - \frac{2b}{a}(L_2 - L_1) \right\} F$$

$$= \left\{ L_1 - \frac{b}{a}(L_2 - L_1) \right\} F$$

Average Resource Cost per User This is the total diversion and suppression cost, averaged over all users; i.e.,

AVERAGE RESOURCE COST

$$= \left\{ L_1 - \frac{a(1-b)}{(1+a)^r} - \frac{b}{a}(L_2 - L_1) \right\} F$$

RESULTS FROM MODEL

Tabulation of Model Outputs

The model was used to produce user reactions under a range of conditions, shown in Tables 1 and 2.

Table 1 is based on a trip cost elasticity of -1, and a value of time of $5 per hour. These figures are typical of domestic air passengers travelling on business.

Table 2 is based on a trip cost elasticity of -2, and a value of time of $1.25 per hour. These are typical of domestic air passengers on leisure trips.

In both cases, a value of 2 is used for the parameter n, which determines the range of the time diversion cost distribution.

In both tables, the conditions considered are:

i) trip costs F: $20, $50, $200
ii) time diversion to avoid peak H: 1, 2, 4 hours
iii) peak charge T: $2, $5, $10, $20.

For each condition, the tabulations show:

i) the proportion of peak users who stay, divert and suppress.
ii) the cost per user for each of the three groups.
iii) the average cost per user; i.e., the total cost resulting from the peak charge, averaged over all users.
iv) the average resource cost per user, i.e. the total resource cost averaged over all users. The resource cost consists of the suppression cost and the diversion cost, but not the peak charge itself, which is simply a transfer payment. It is this cost which would have to be compared with the cost of additional capacity, if that were the alternative to the use of a peak pricing scheme.

Reactions to Peak Charge

Broadly, the results produced by the model for the conditions considered show a wide range of reaction.

TABLE 1: TABULATION OF REACTIONS TO PEAK CHARGE - PASSENGERS WITH ELASTICITY r = 1, TIME VALUE $5/hr. RANGE PARAMETER n = 2 (AIR BUSINESS PASSENGERS)

Trip Cost F $	Divert Time H hours	Peak Charge T $	Proportions: Stay %	Proportions: Divert %	Proportions: Sup-press %	Costs by Group: Stay $/pass	Costs by Group: Divert $/pass	Costs by Group: Sup-press $/pass	Total Cost $/pass	Re-source Cost $/pass
20	1	2	73	19	8	2.0	1.0	0.9	1.7	0.3
20	1	5	40	45	15	5.0	2.4	2.0	3.4	1.4
20	1	10	0	81	19	-	4.7	2.9	4.3	4.3
20	1	20	0	81	19	-	4.7	2.9	4.3	4.3
20	2	2	82	9	9	2.0	1.0	0.9	1.8	0.2
20	2	5	60	22	18	5.0	2.4	2.2	3.9	0.9
20	2	10	33	41	26	10.0	4.7	3.8	6.2	2.8
20	2	20	0	69	31	-	8.8	5.2	7.7	7.7
20	4	2	86	5	9	2.0	1.0	1.0	1.9	0.1
20	4	5	70	11	19	5.0	2.4	2.3	4.2	0.7
20	4	10	50	20	30	10.0	4.7	4.1	7.2	2.2
20	4	20	25	35	40	10.0	8.8	6.8	10.8	5.8
50	1	2	77	20	3	2.0	1.0	0.9	1.8	0.2
50	1	5	46	48	7	5.0	2.5	2.2	3.6	1.3
50	1	10	0	91	9	-	4.8	3.1	4.7	4.7
50	1	20	0	91	9	-	4.8	3.1	4.7	4.7
50	2	2	86	10	4	2.0	1.0	1.0	1.9	0.1
50	2	5	68	24	8	5.0	2.5	2.3	4.2	0.6
50	2	10	42	45	13	10.0	4.8	4.2	6.9	2.7
50	2	20	0	84	16	-	9.4	5.9	8.9	8.9
50	4	2	91	5	4	2.0	1.0	1.0	1.9	0.1
50	4	5	79	12	9	5.0	2.5	2.3	4.5	0.5
50	4	10	62	23	15	10.0	4.8	4.5	8.0	1.8
50	4	20	36	42	22	20.0	9.4	7.8	12.9	5.7
200	1	2	79	20	1	2.0	1.0	0.7	1.8	0.2
200	1	5	49	49	2	5.0	2.5	1.1	3.7	1.3
200	1	10	0	98	2	-	5.0	3.3	4.9	4.9
200	1	20	0	98	2	-	5.0	3.3	4.9	4.9
200	2	2	89	10	1	2.0	1.0	0.9	1.9	0.1
200	2	5	73	25	2	5.0	2.5	1.9	4.3	0.7
200	2	10	47	49	4	10.0	5.0	4.4	7.4	2.6
200	2	20	0	95	5	-	9.8	6.4	9.7	9.7
200	4	2	94	5	1	2.0	1.0	0.9	1.9	0.1
200	4	5	86	12	2	5.0	2.5	2.2	4.6	0.4
200	4	10	72	24	4	10.0	5.0	4.6	8.6	1.4
200	4	20	45	48	7	20.0	9.8	8.7	14.4	5.3

TABLE 2: TABULATION OF REACTIONS TO PEAK CHARGE - PASSENGERS WITH ELASTICITY r = 2, TIME VALUE $1.25/hr, RANGE PARAMETER n = 2 (AIR LEISURE PASSENGERS)

Trip Cost F	Divert Time H	Peak Charge T	Proportions			Costs by Group			Total Cost	Re-source Cost
			Stay	Divert	Sup-press	Stay	Divert	Sup-press		
$	hours	$	%	%	%	$/pass	$/pass	$/pass	$/pass	$/pass
20	1	2	16	73	11	2.0	1.0	0.7	1.1	0.8
20	1	5	0	89	11	-	1.2	0.8	1.2	1.2
20	1	10	0	89	11	-	1.2	0.8	1.2	1.2
20	1	20	0	89	11	-	1.2	0.8	1.2	1.2
20	2	2	50	36	14	2.0	1.0	0.9	1.5	0.5
20	2	5	0	80	20	-	2.3	1.5	2.1	2.1
20	2	10	0	80	20	-	2.3	1.5	2.1	2.1
20	2	20	0	80	20	-	2.3	1.5	2.1	2.1
20	4	2	66	18	16	2.0	1.0	0.9	1.6	0.3
20	4	5	32	40	28	5.0	2.3	2.0	3.1	1.5
20	4	10	0	67	33	-	4.3	2.7	3.8	3.8
20	4	20	0	67	33	-	4.3	2.7	3.8	3.8
50	1	2	18	77	5	2.0	1.0	0.7	1.2	0.8
50	1	5	0	95	5	-	1.2	0.9	1.2	1.2
50	1	10	0	95	5	-	1.2	0.9	1.2	1.2
50	1	20	0	95	5	-	1.2	0.9	1.2	1.2
50	2	2	56	38	6	2.0	1.0	0.9	1.5	0.4
50	2	5	0	91	9	-	2.4	1.6	2.3	2.3
50	2	10	0	91	9	-	2.4	1.6	2.3	2.3
50	2	20	0	91	9	-	2.4	1.6	2.3	2.3
50	4	2	74	19	7	2.0	1.0	0.9	1.7	0.3
50	4	5	41	45	13	5.0	2.4	2.1	3.4	1.4
50	4	10	0	83	17	-	4.7	3.0	4.4	4.4
50	4	20	0	83	17	-	4.7	3.0	4.4	4.4
200	1	2	20	79	1	2.0	1.0	0.8	1.2	0.8
200	1	5	0	99	1	-	1.2	0.4	1.2	1.2
200	1	10	0	99	1	-	1.2	0.4	1.2	1.2
200	1	20	0	99	1	-	1.2	0.4	1.2	1.2
200	2	2	59	39	2	2.0	1.0	0.9	1.6	0.4
200	2	5	0	98	2	-	2.5	1.9	2.5	2.5
200	2	10	0	98	2	-	2.5	1.9	2.5	2.5
200	2	20	0	98	2	-	2.5	1.9	2.5	2.5
200	4	2	78	20	2	2.0	1.0	0.9	1.8	0.2
200	4	5	47	49	4	5.0	2.5	3.3	3.7	1.3
200	4	10	0	95	5	-	4.9	3.4	4.8	4.8
200	4	20	0	95	5	-	4.9	3.4	4.8	4.8

These reactions perhaps indicate a greater sensitivity to the peak charge than might be expected. However, in interpreting these results, it is important to recognise that the model is concerned with long term reactions; i.e. it assumes that the charges are well known when the trip is being planned, so that appointments can be made with these in mind. It is not a model of the reaction of a passenger at an airport, suddenly told he can either pay $5 or be delayed 2 hours.

Business Passengers On a $50 trip, for passengers travelling on business and faced with a $5 peak charge or a 2 hour time diversion, the model predicts that 68% will pay the peak charge, 24% will travel off peak, and 8% will not travel. If this charge is increased to $10, only 42% continue to travel in the peak, while if the charge is only $2, 86% pay the charge and travel in peak.

If the time diversion is 1 hour, only 46% travel in the peak, and 48% travel off peak, while if the time diversion is 4 hours, 79% are prepared to pay the peak charge, and only 12% travel out of the peak.

The level of trip cost also has an influence on reaction. For a $5 peak charge and a 2 hour time diversion, 60% pay the peak charge for a $20 trip cost, and 73% for a $200 trip, while 68% pay for a $50 trip. This implies that the longer the trip, the less sensitive the passenger to the peak charge.

Leisure Passengers On the same $50 trip with $5 peak charge and 2 hour time diversion, the model predicts leisure passengers to be much more sensitive to the peak charge than business passengers. 91% of passengers travel off peak to avoid the charge, and the remaining 9% find the trip no longer worthwhile. None travel during peak.

For higher peak charges, these reactions are unchanged, as there are no peak passengers left for the charge to act upon. At a peak charge of $2, 56% continue to travel in the peak, 38% travel off peak, and 6% do not travel.

At a time diversion of 4 hours, 41% of leisure passengers are prepared to pay a $5 charge to travel at their preferred time, while for a 1 hour time diversion, no passengers are prepared to pay $5 to travel at their preferred time.

As for business passengers, the reaction to the peak charge is higher on the low cost trips and lower on the high cost trips.

Costs Resulting from Peak Charge

Costs to Users who Stay Those users who elect to travel in the peak and pay the peak charge suffer a cost equal to the peak charge.

Costs to Users who Divert Those users who accept a time diversion suffer a cost which is slightly less than half the peak charge, for peak charge levels up to the maximum of the range of time diversion costs. For charges above this level, the average cost suffered is slightly less than half the maximum time diversion cost.

Costs to Users who Suppress The users who elect not to travel at all are the low surplus, high time diversion cost users. The cost to them is their loss of surplus, and the average value of this loss ranges from half the peak charge, for very low levels of peak charge, to about one third of the peak charge, for a peak charge equal to the maximum time diversion cost. For peak charges above this level, the cost to passengers does not further increase.

<u>Average Cost to Users</u> The cost to users, averaged over the three groups, ranges from a little less than a peak charge, for very small values of peak charge, to about half of the peak charge, when this charge is equal to the maximum time diversion cost. Further increases in the peak charge have no effect on the cost to users.

<u>Average Resource Cost</u> This is the cost of time diversion and loss of surplus, averaged over all users. It ranges from a negligible amount, when most users travel in the peak and pay the charge, to a value equal to the average cost to users, when no users travel in the peaks.

DISCUSSION OF MODEL

Assumptions Used

There are three main assumptions in the model. These are discussed briefly, in the light of the results obtained in the previous section.

It is important to recognise that the most appropriate assumptions in each case may well depend on the particular transport facility being considered. The discussion given is directed towards the use of airports. Analogous arguments could be made with respect to other facilities.

<u>Constant Elasticity of Demand</u> This is a commonly made assumption, and is probably quite satisfactory. In fact the results show that, over the range of conditions considered, the maximum suppression was 40%. Thus the assumption of constant elasticity need only apply from the unrestricted demand level to a level 40% lower than this level,providing that it does not change abruptly below this level.

Uniform Distribution of Time Diversion Cost There is no firm basis for this assumption. The distribution should have a lower limit of O, as it is highly likely that there will be some users who are not concerned with just when they travel.

The upper limit is determined by the range parameter n, and a value of 2 was used in the tabulations in the previous section. This leads to a maximum time diversion cost of 2 U $ per hour of time diversion, where U is the users value of time. It is possible that a higher value of n would be more appropriate. This would tend to reduce the proportion of diverters, and to increase the proportion of stayers and suppressers.

It is possible that there are fewer users with extreme values of the time diversion cost than with central values, so that it might be argued that the distribution should be more like a truncated normal distribution. It would be of some interest to determine the effect on the results of the use of a distribution of this form.

Independence of Surplus and Time Diversion Cost There was no data to support this assumption. The basis for it is:

i) it cannot be argued easily that S and D are either positively correlated or negatively correlated

ii) it is possible to give examples of trips which have
- high S and high D
- high S and low D
- low S and high D
- low S and low D

The results in the previous section seem generally to support this assumption. It might, however, be argued that the proportion of high D low S trips is somewhat high. This results in the 9% suppressers and 0% stayers for leisure passengers with $50 peak charge. It would perhaps be more reasonable to have more stayers and fewer suppressers under these conditions.

If so, this would suggest that some small amount of positive correlation of S and D is appropriate, together with a somewhat higher value of the range parameter n.

However, a change of this type is probably not justified without some sort of basis from observed user behaviour.

Model Validation

When the model was developed, the only data available for use in validation was the effect of the airlines' recently introduced off-peak fares.

An attempt was made to use this data for validation, and to give an indication of the best value for the range parameter n. The data was, however, very noisy, and statistically significant results were not obtained, although the indication was that a somewhat higher value of n might be more appropriate.

It would add considerably to the value of this model if this validation could be extended, either by a survey, or by collection of more detailed information on passenger traffic.

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APPENDIX

This appendix gives the evaluation of the integrals used in the calculation of the proportions of users who stay, transfer, and suppress, and the costs associated with each. The numbers refer to the numbers in the text.

The notation used is:

S	=	trip surplus
D	=	time diversion cost
F	=	generalised trip cost
T	=	peak charge
C	=	maximum time diversion cost
a	=	T/F
b	=	T/C
r	=	elasticity of trip cost

$$L_1 = \int_0^T \frac{F^{r-1}}{(F+S)^r}\, dS = \begin{cases} \dfrac{1-(1+a)^{1-r}}{r-1} & r \neq 1 \\ \ln(1+a) & r = 1 \end{cases}$$

$$L_2 = \int_0^T \frac{F^{r-2}}{(F+S)^{r-1}}\, dS = \begin{cases} \dfrac{1-(1+a)^{2-r}}{r-2} & r \neq 2 \\ \ln(1+a) & r = 2 \end{cases}$$

(1) PROPORTION OF STAYERS

$$= \int_{S=T}^{\infty} \int_{D=T}^{C} \frac{rF^r}{C(F + S)^{r+1}} \, dD \, dS$$

$$= \frac{C - T}{C} \int_{S=T}^{\infty} \frac{rF^r}{(F + S)^{r+1}} \, dS$$

$$= \left(1 - \frac{T}{C}\right) \frac{F^r}{(F + T)^r}$$

$$= \frac{1 - b}{(1 + a)^r}$$

(2) PROPORTION OF DIVERTERS

$$= \int_{D=0}^{T} \int_{S=D}^{\infty} \frac{rF^r}{C\ (F + S)^{r+1}} \, dS \, dD$$

$$= \frac{F}{C} \int_{D=0}^{T} \frac{F^{r-1}}{(F + D)^r} \, dD \qquad (r \neq 0)$$

$$= \frac{b}{a} \quad L_1$$

(3) PROPORTION OF SUPPRESSERS

$$= \int_{S=0}^{T} \int_{D=S}^{C} \frac{rF^r}{C\,(F+S)^{r+1}}\, dD \; dS$$

$$= \int_{S=0}^{T} \frac{C-S}{C} \; \frac{rF^r}{(F+S)^{r+1}}\, dS$$

$$= \int^{T} \left(\frac{rF^r}{(F+S)^{r+1}} - \frac{(r-1)F^r}{C(F+S)^r} - \frac{F^r}{C(F+S)^r} + \frac{rF^{r+1}}{C(F+S)^{r+1}} \right) dS$$

$$= (1 + \frac{F}{C}) \; (1 - \frac{F^r}{(F+T)^r}) - \frac{F}{C} \; F^{r-1} \; (\frac{1}{F^{r-1}} - \frac{1}{(F+T)^{r-1}}) - \frac{F}{C} L_1$$

$$= (1 + \frac{b}{a}) \; (1 - \frac{1}{(1+a)^r}) - \frac{b}{a} \; (1 - \frac{1}{(1+a)^{r-1}}) - \frac{b}{a} L_1$$

$$= 1 - \frac{1-b}{(1+a)^r} - \frac{b}{a} \; L_1$$

(4) TOTAL DIVERTER COST

$$= \int_{D=0}^{T} D \int_{S=D}^{\infty} \frac{rF^{r}}{C(F + S)^{r+1}} \, dS \quad dD$$

$$= \frac{F^2}{C} \int_{0}^{T} \frac{DF^{r-2}}{(F + D)^{r}} \, dD$$

$$= \frac{F^2}{C} \int_{0}^{T} \left(\frac{F^{r-2}}{(F + D)^{r-1}} - \frac{F^{r-1}}{(F + D)^{r}} \right) dD$$

$$= \frac{b}{a} \left(L_2 - L_1\right) F$$

(5) TOTAL SUPPRESSER COST

$$= \int_{S=0}^{T} S \int_{D=S}^{T} \frac{rF^r}{C\,(F+S)^{r+1}}\, dD \quad dS$$

$$= \int_{0}^{T} \frac{(C-S)\,S\,r\,F^r}{C\,(F+S)^{r+1}}\, dS$$

$$= \int_{0}^{T} \left(\frac{rF^r}{(F+S)^r} - \frac{rF^{r+1}}{(F+S)^{r+1}} - \frac{rF^r}{C(F+S)^{r-1}} + \frac{2rF^{r+1}}{C(F+S)^r} - \frac{rF^{r+2}}{C(F+S)^{r+1}} \right) dS$$

$$= \int_{0}^{T} \left((1+\frac{F}{C}) \frac{-rF^r}{(F+S)^{r-1}} + (1+\frac{2F}{C}) \left(\frac{(r-1)F^{r-1}}{(F+S)^r} + \frac{F^{r-1}}{(F+S)^r} \right) - \frac{F}{C} \left(\frac{(r-2)F^{r-2}}{(F+S)^{r-1}} + \frac{2F^{r-2}}{(F+S)^{r-1}} \right) \right) F dS$$

$$= \left((1+\frac{b}{a}) \left(\frac{1}{(1+a)^r} - 1 \right) + (1+\frac{2b}{a}) \left(1 - \frac{1}{(1+a)^{r-1}} + L_1 \right) - \frac{b}{a} \left(1 - \frac{1}{(1+a)^{r-2}} + 2L_2 \right) \right) F$$

$$= \left(L_1 - \frac{a(1-b)}{(1+a)^r} - \frac{2b}{a} (L_2 - L_1) \right) F$$