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PROJECT APPRAISAL UNDER UNCERTAINTY - PROBABILISTIC COST BENEFIT ANALYSIS

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Abstract:

In most project evaluations the cost and particularly the benefit parameter values carry a high degree of uncertainty or subjective judgement. Nevertheless it is general practice for evaluation to use single value estimates of parameters, and for point estimates to be provided for the generated decision criteria (the benefit cost ratio or net present value).

This paper shows how risk and uncertainty can be simply incorporated into cost-benefit analysis through the application of probability theory. A simple computer programme for this is developed and a hypothetic road project is analysed for illustrative purpose.

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PROBABILISTIC COST BENEFIT ANALYSIS*

INTRODUCTION

> "We plan in a world of limited knowledge, a world in which facts are probabilistic and values debatable. Under such circumstances 'correct' decisions do not exist. The merit of a decision can only be appraised by values held individually or in a collectivity, but such values .... are not verifiable. In such a situation the goal for decision-making should be increasing the degree of assurance (of decision makers and clients) that the choice made was at least as reasonable or more reasonable than any other alternative. This goal is best attained by bringing to bear on every decision the greatest amount of relevant information concerning the ramifications of all alternatives."
>
> (Davidoff and Reiner 1962)

Uncertainty pervades every aspect of economic evaluation - yet it is rarely considered explicitly. This paper aims to demonstrate that it is comparatively simple to take into account the uncertainty surrounding parameter values, and that this enables the analyst to provide the decision maker with additional detail for subsequent trade offs. The shortcomings of traditional deterministic (point estimate) techniques and the advantages of probabilistic techniques are discussed, and an example of probabilistic evaluation is presented. There has been some

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discussion in the economic literature about whether the expected value of a project alone or the expected value plus some measure of the risk or uncertainty (eg probabilistic data) associated with the project are the relevant criteria for measuring the economic value of a public project. Dasgupta and Pearce (1972), for example, argue that the expected value alone is sufficient. Appendix 1 presents a short guide to that discussion and argues that the risk associated with a project will often be a relevant decision criterion in transport projects. For this reason in reporting the results of the case study the cumulative probability distribution of the outcome of the project is given some emphasis.

This paper should not be necessary. As Klausner (1969) noted over a decade ago "the limitations of traditional methods of dealing with uncertainty in capital investment analysis have been recognised for some time". However, this knowledge does not seem to have strongly influenced practice by public sector transport economists in Australia. Outside of agricultural economics (Cassidy and Gates 1977) and hydraulic engineering (Howell 1977a, 1977b) few Australian public sector evaluations have explicitly accounted for risk or uncertainty.(1)

INVESTMENT GOALS AND EVALUATION CRITERIA

Broadly speaking the goal of public sector decision making is to maximise national welfare. National welfare, however is a high level abstraction and is not, and probably never will be, capable of representation by a single objective function. Instead multiple goals relating to the economic, financial, social, distributional, environmental and other outcomes are inherent in all but the most simple public investment decision context.

However, alternative criteria may, and often do, point to different conclusions. The reconciliation of such conflicts is properly the function of the decision maker, not the analyst. Thus it is incumbent on the analyst to communicate the results of the various studies (economic, environmental, political, etc) to the decision makers in a manner which facilitates the trade off process.

Economic and Other Criteria - The Trade Off Process

We suggest that, to date in Australia, the results of public sector economic evaluations have not generally been presented in a manner conducive to aiding decision makers in this trade off process. This is due partly to the traditional deterministic approach used and partly to a failure in communication on the part of analysts. It is with the former that we are concerned in this paper - although we do touch on effective communication.

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1 Exceptions include Clark and Segal 1977; W.D. Scott 1976; BIE 1980 (forthcoming).

The limited usefulness of the deterministic approach to economic evaluation in providing information that facilitates the trade off process is probably best illustrated by reference to an actual study (W.D. Scott 1978). In one of the many analyses of options for upgrading Sydney's airport facilities, an evaluation of two alternative sites, N1 and SW4, showed the net present value (NPV) of N1 to be $26 million lower than that of site SW4. The social and environmental differences between the sites were considerable but, confronted with a difference in NPV of $26 million, the economic case might well have been over-powering - had the traditional deterministic approach been adopted.

However, because there were major uncertainties associated with a number of variables a crude probabilistic analysis was undertaken. The impact on net present value of uncertainty surrounding inaccuracies in data inputs (the most objective and measurable inputs of all - land costs, building costs, etc), was examined. The result of this analysis provided very revealing information; at a 95 per cent confidence level the range for the net present value of site SW4 compared with N1 was $26 million ± $49 million. Leaving aside the uncertainties associated with all other factors, this suggested that the cost differential between the sites was not statistically significant - a far cry from the implications of a single point estimate of NPV of $26 million. Presented with this new information, the decision makers would be better placed to account for social, environmental and other differences between the alternatives not measured in monetary terms.

DEFICIENCIES IN DETERMINISTIC EVALUATIONS

In the usual approach to project evaluation, point estimates representing the "most likely" (usually the modal, but sometimes taken as the mean or even median) values are made for each element entering the computation of cost and benefit streams; point estimates of the total discounted cost and benefit streams are computed and the 'expected' value of the decision criterion (generally net present value or benefit cost ratio) is derived.

In general most of the input parameters in public sector economic analysis carry some degree of uncertainty or subjective judgement. Nonetheless, this is rarely acknowledged explicitly, and indeed by reporting point estimates to four, six or more significant figures the analyst frequently implies a quite fallacious degree of exactitude.(1) With such apparent accuracy the decision maker can be forgiven for placing too much credence on the figures in the evaluation.

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1 For example, BIE (1975) reported air passenger forecasts for the year 2000 to six significant figures, 14 282.4 million passenger movements (Table 2.1); domestic air freight forecasts for the year 2000 were reported to five significant figures, eg 148.59 thousand tonnes (Table C.11); forecast vehicle kilometres of travel in the region in 1991 were reported to seven significant figures, eg 1 644 502 kilometres (Table G.3).

There are several major problems with a "point estimate" approach:

- The decision maker, usually being remote from the analysis, has no "feel" for the accuracy or significance of the data and is not in a position to interpret sensitivity tests or the analysts' qualifications to the results.
- No information on likelihood of failure is given. Particularly where disruption to a service would be severe (for example the collapse of the Tasman bridge) such data may be critical to understanding the problem.
- No information is available on whether differences between expected outcomes are statistically significant and hence whether differences between the net present values of alternatives are at all relevant in subsequent trade offs.
- There is no check against consistent errors of bias. As will be discussed in a later section, there is some evidence of consistent bias in public sector evaluations particularly in demand forecasting.
- Where there are inputs to an analysis from multiple sources there is difficulty in ensuring consistent treatment of uncertainty and a danger of multiple conservative adjustment to the same data sets as successive analysts build in their own safety factors.
- Single value techniques are unable to take into account the effect of skewed probability distributions of variables or interaction between variables.

Traditional Treatment of Uncertainty

Analysts are generally aware of these problems, even though the exaggerated "accuracy" in reporting, noted early, might suggest otherwise. Traditionally, uncertainty has been dealt with in one of the following ways:

- Introducing conservatism into cost and benefit estimates (but evidence suggests bias, particularly in respect of benefits, is not usually in the direction of conservation!).
- Including sensitivity analyses of key variables; essentially this technique identifies how sensitive the output is to changes or errors in particular variables. Table 1 from McNamara (1978) is a typical example. As is clear from this table, however, the decision maker has no means of gauging the likelihood of the occurrence of alternative values of a given variable or of combinations of alternative values of several key variables.
- Adjusting (upwards) the discount rate to reflect a risk premium; this implies uncertainty is compounding over time, which is rarely justified.

TABLE 1: PETRIE ROUTE - SENSITIVITY COMPARISONS OF ANNUAL ECONOMIC BENEFITS

DESCRIPTION	ANNUAL BENEFITS AND COSTS ($'000)							
Land Use Strategy	PETRIE B		PETRIE C		PETRIE D		PETRIE E	
Public Transport Service	Rail-Bus		Rail-Bus		Rail-Bus		Rail-Bus	
Parameter Values	'Base" (b)		'Fuel Increase' (c)		'Vehicle Operating Cost Increase' (d)		'Extreme' (e)	
Discount Rates(a)	Low	Base	Low	Base	Low	Base	Low	Base
Benefits								
User Benefits	+ 740	+ 740	+ 800	+ 800	+1480	+1480	+1640	+1640
Costs								
- Bus								
. Capital	+ 60	+ 80	+ 100	+ 120	+ 120	+ 160	+ 120	+ 160
. Operating	+ 140	+ 140	+ 220	+ 240	+ 240	+ 240	+ 260	+ 260
- Rail								
. Capital	-1400	-1820	-1400	-1820	-1400	-1820	-1400	-1820
. Operating	-1080	-1080	-1080	-1080	-1080	-1080	-1080	-1080
Accident Costs	+ 80	+ 80	+ 40	+ 40	+ 120	+ 120	+ 160	+ 160
TOTAL NET BENEFITS	-1460	-1860	-1320	-1720	- 520	- 900	- 300	- 680

SOURCE: Based on Table 9.9, McNamara (1978)

a Discount rates for low case - 7 per cent public transport and 10 per cent private: base case - 10 per cent public transport and 15 per cent private.

b Base case parameters represent 'most likely' scenarios.

c Assumes fuel prices increase by 100 per cent in real terms over 1976/7 prices. All other paramaters at 1976/7 values.

d Overall vehicle operating costs increase by 100 per cent in real terms over 1976/7 prices (which is equivalent to a 370 per cent real increase in fuel prices). All other parameters at 1976/7 values.

d An "Extreme" case combining the following parameter adjustments: fuel price up by 250% in real terms; fares reduced by 50% in real terms; parking cost up by 100% in real terms; public transport travel times down by 10%; private car travel times up by 25%.

- Arbitrarily shortening the assumed project life; the "pay back period" approach being the archetype of this method. This method implicitly assumes benefits and costs are certain during the pay back period and are so uncertain thereafter that they should be ignored.

- Ad hoc qualification of the inputs, the outputs or the conclusions.

UNCERTAINTY IN ECONOMIC EVALUATION

Defining Uncertainty

Precise definition is difficult. Solely to illustrate what we encompass under the term, we suggest the following categorisation:

- Uncertainty due to the stochastic nature of the phenomenon in question - for example the return period of a flood.

- Statistical uncertainty due to the limited number of observations from which parameters are estimated - for example estimates of present demand for a product.

- Professional uncertainty arising from incomplete information concerning the underlying process and its probabilistic representation - for example projections of future demand, projections of technological change and so forth.

Clearly all three types of uncertainty may be present in a single parameter - for example projections of flood exceedence in a particular locality in remote areas of Australia may have uncertainty due to the stochastic nature of the occurrence of monsoonal rains, uncertainty due to the limited record of rainfall data and uncertainty due to lack of knowledge for that region of infiltration rates, overland flow rates, evaporation and so forth.

The Source and Level of Uncertainty in Project Evaluation

Striking examples of evident discrepancies between project evaluation and outcome spring easily to mind. Melbourne's $145 million West Gate Bridge is perhaps the best recent example. A catastrophic accident, industrial disputes and design changes all helped push real cost levels up, while changes in community attitude to freeways (leading to deletion of the major freeways linking the bridge to the southern and south eastern suburbs), a $30 million Federal grant for the Johnson Street bridge, a dramatic shortfall in metropolitan population growth from that predicted and changes in urban planning strategy have served to cut traffic (and hence economic and financial benefits) to about half that projected by the Country Roads Board as late as 1977 (Age 15.11.79, p 20) and to less than one-fifth that originally forecast in the West Gate Bridge Authority's evaluation studies in the late 1960s.

Such a dramatic example of evaluation failure, of course, is the exception. There is nevertheless plenty of evidence of the uncertainty surrounding cost estimation and demand forecasting.

Various US studies (eg Hufschmidt and Gerin (1970) and Merewitz (1973)) suggest that, while there is little evidence of systematic bias in cost estimation,[1] the standard deviation of the distribution of estimating errors can be high. In an analysis of 182 inflation adjusted project cost estimates by the US Corps of Engineers, Hufschmidt and Gerin found the standard deviation of the estimating error distribution to be 43 per cent.

Table 2 summarises the findings of a coarse comparison of preliminary cost estimates with final (as constructed) costs for 86 Australian capital works projects built between 1972 and 1978. In the analysis all cost estimates were adjusted to reflect price level changes,[2] and the ratios of final cost to estimated cost were examined for three price ranges as detailed in the table.

TABLE 2: COMPARISON OF PROJECT COST ESTIMATES WITH ACTUAL COSTS

No of Projects	Project Scale	Final Cost ÷ Estimate[a]	
		Mean	SD
40	\$100 000 - \$500 000	.998	0.25
9	\$500 000 - \$1 million	1.09	0.21
37	\$1 million - \$20 million	0.92	0.28

(a) For the first two categories of project scale the estimate used was the 'pre-tender' estimate based on contract drawings and bill of quantitites. For projects over \$1 million the estimate was the preliminary estimate given to a parliamentary works committee.

Examination of the cost records for the projects under \$1 million suggested the major cause of variation was due to client requested changes in scope. This accords with the analysis by Robinson (1972) of 500 capital works projects; in a sample of 21 projects above \$100 000, 1850 project variations were issued once construction had started - with 206 variations to one project.

1 Summers (1967) provides some evidence to the contrary - in his analysis of estimated versus actual costs, he found that the mean ratio of inflation adjusted actual costs to estimated costs was 1.79.

2 We note that for projects over \$1 million, for which the construction time was generally more than two years, no data was available on cash flow - and our assumption of even cash flow for purposes of price level adjustment is a possible source of error.

Similarly in regard to forecasting Maridakis and Hibon (1979) and Liebling and Russell (1969) demonstrate the considerable scope for error in time series and more sophisticated econometric forecasting techniques. In airport evaluation, for example, the results are clearly highly dependent on the accuracy of passenger and freight forecasts. BTE (1975) produced the following figure, Figure 1, to illustrate the effect of different assumptions concerning air passenger growth through Brisbane airport. Depending on the model used, the BTE report shows that by 1980, six years from the forecast date, the "most likely" predicted passenger throughput would be between 2 million and 6 million! To emphasise further the uncertainty inherent in forecasting, we have included in Figure 1 the official forecasts made at different times since 1967, and the actual passenger movements up to 1979/80.

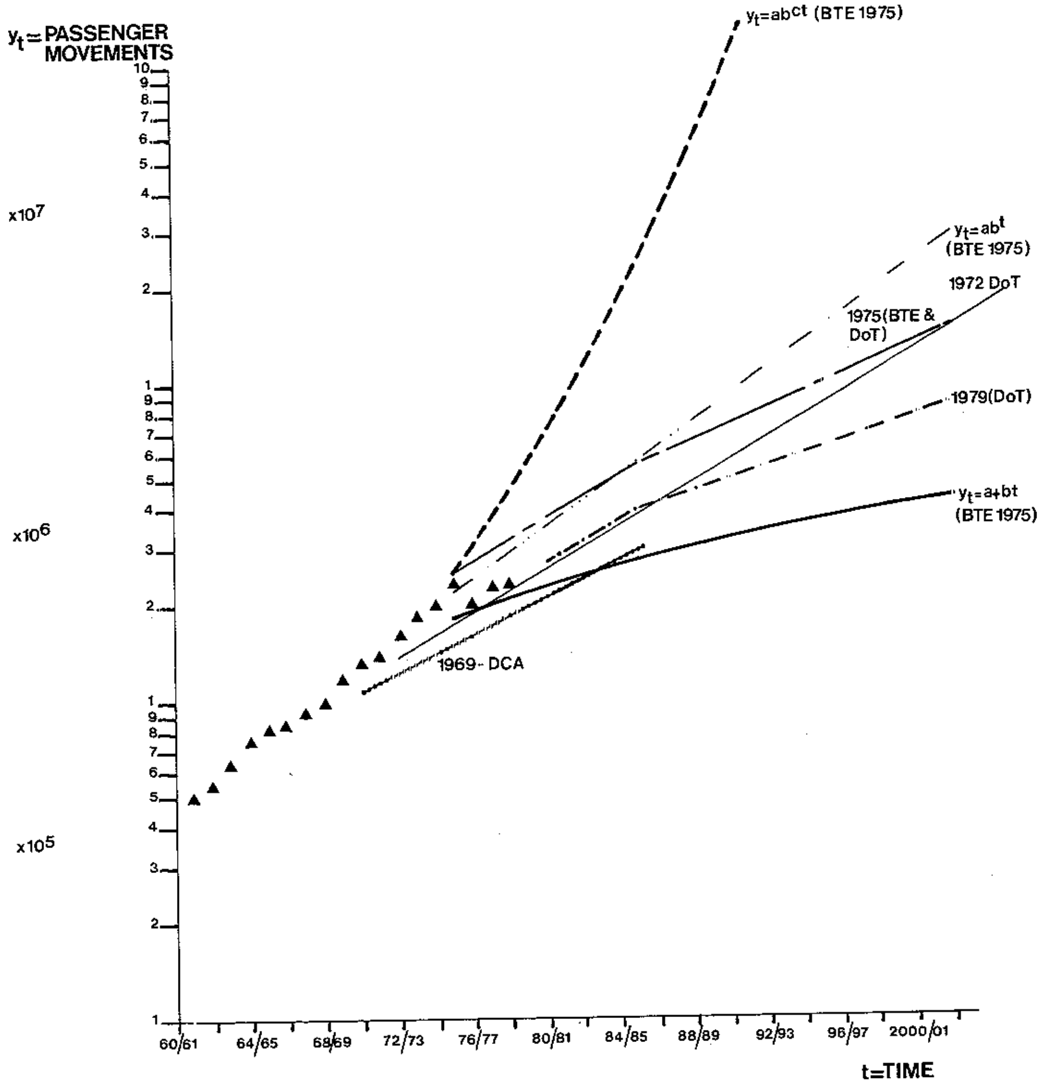
Webster (1975) in a similar comparison of US Federal Aviation Administration forecasts over time with actual passenger movements showed similar experience with forecasting. De Naufville (1976), Table 3, showed that the likely errors in air traffic forecasts increased by about 3 per cent for each year further ahead.

TABLE 3: AVERAGE ERRORS IN THE FORECASTS OF AIR TRAFFIC BY THE FAA 1958-1971

Nature of Traffic	Type of Traffic	Per Cent Error which is Exceeded Half the Time after each Year				
		One Year	Two Years	Three Years	Four Years	Five Years
International	Passengers	8.0	10.9	14.0	14.0	15.1
	Pax miles	7.2	10.9	13.8	15.7	16.7
	Average	7.6	10.9	13.9	14.9	15.9
Domestic	Passengers	2.9	6.2	11.2	14.1	18.3
	Pax miles	2.9	6.0	10.9	14.5	18.3
	Average	2.9	6.1	10.9	14.3	18.3

Source: De Neufville (1976).

As we move to evaluation of major transportation projects and programs the potential sources of error compound. Mackinder (1979) lists some of the major error sources (Table 4).



Source: BTE (1975), Department of Transport

Notes : BTE 1975: These were included in the BTE report to illustrate the implications of various forecasting models.
1975 (BTE and DOT): This was the actual econometric model chosen in the BTE report.

FIGURE 1: COMPARISON OF ACTUAL AND FORECAST PASSENGER MOVEMENTS AT BRISBANE AIRPORT

TABLE 4: SUMMARY OF POSSIBLE ERROR SOURCES

	Base Year Model		Forecast Year	
	Measurement Error	Specification Error	Predictive Error	Specification Error
Trip production model				
Planning data	X		X	
Zonal car ownership	X		X	X
Zonal income	X	X	X	
Cross classification of households	X	X		X
Trip rates	X	X		X
Trip attraction model				
Planning data	X		X	
Attraction rates	X	X		X
External trip ends	X		X	
Trip distribution				
Zone to zone costs	X	X	X	X
Zone to zone trips	X	X		X
Modal split				
Zone to zone costs by mode	X	X	X	X
Zone to zone trips by mode	X	X		X
Commercial vehicle trips	X		X	X
Assignment				
Link description	X		X	
Minimum cost routings		X		X
Vehicle occupancy	X			X
Capacity restraint	X	X	X	X

Source: Mackinder (1978), Table 1.

There are many significant areas of professional uncertainty. The difficulties in reconciliating the social time preference rate and social cost of capital lead to uncertainities in the appropriate discount rate - with a consensus of professional opinion putting it between about 7 per cent and 12 per cent. The valuation of private travel time is fraught with uncertainty - with professional opinion ranging from zero to 200 per cent or more of average earnings. Changes in vehicle operating cost and accident reductions consequent on project improvements are equally uncertain (eg McDonald, 1974; Jorgensen, 1978).

Finally, as alluded to in the case of the West Gate Bridge, there is the uncertainty arising from changes in the "rest of the world" - government policy, planning strategies, new technology, international events (eg Arab-Israeli war 1973, Iran 1979).

Because transportation projects generally have long gestation periods, are long lived and include a number of the potential sources of uncertainty discussed above, we believe there is a strong case for taking uncertainty explicitly into account.

Probabilistic Evaluation

Risk analysis and probabilistic evaluation techniques are certainly not new, and have been applied to corporate investment analyses for almost a quarter of a century; a prolific literature on risk analysis in capital investment has developed since David Hertz's seminal paper in 1964 (Hertz, 1964).

The diffusion of such ideas to the public sector has been slower, even though stochastic techniques have long been applied in relation to agricultural investment and to flood control works.

In essence the probabilistic approach requires the analyst to generate probability distributions(1) for parameters which have a degree of risk or uncertainty, identify correlations between parameters and combine costs and benefits to derive a probability distribution for the decision criterion, eg NPV.

Perhaps the most important result of a move to probabilistic evaluation is that the alternatives presented for evaluation will almost certainly be modified. Probabilistic evaluation should encourage the design of options or alternatives which provide for reconsideration after a certain period and the possibility of a change in direction, or which are capable of performing adequately under a range of options (Harrison, 1974). Haefner (1979) demonstrates the benefits of this concept by combining Bayesian statistical techniques and Markovian decision theory in a cost

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1 The generation of probability density functions for the various parameters is beyond the scope of this paper. The reader is referred to Raiffa (1965), to Kaufman and Thomas (eds, 1977) and in particular to the papers in that anthology by Moore and Thomas (1975) and Tveresky and Kahneman (1974).

benefit evaluation of airport development strategies in San Francisco. Given the lengthy planning horizon for all major transportation works, such planning for options seems essential.

Other advantages of the probabilistic approach to evaluation may be summarised as:

- Assumptions concerning uncertainty are made explicit to the decision maker: the analyst is forced to identify explicitly, either on a theoretical or an empirical basis or on the basis of his subjective judgement, the degree of uncertainty relating to significant input parameters. The requirement to develop probability density functions for important variables is a discipline on the analysts to refine their estimates of the more significant variables.

- Limits can be placed on the output of the evaluation - thus giving an appreciation of the statistical significance of the results.

- Sensitivity analysis, involving specification of alternative probability distributions of occurrence of given key variables, can be undertaken.

- The likelihood of complete economic failure of the project can be identified.

- The analyst is forced to give attention to the communication of the meaning of the analysis because the statistical output, unlike the output of deterministic studies, is not apparently self evident.

PROBABILISTIC FORMULATION OF NET PRESENT VALUE

One can undertake probabilistic analysis either using mathematical formulation based on well known properties of statistical distributions, or by using Monte Carlo simulation methods.

Mathematical Formulation

Generally mathematical analysis is feasible with comparatively simple problems where it is reasonable to make simplifying assumptions concerning the shape of the probability density functions. The advantage claimed for such analytical approaches (Hillier 1963; Wagle 1967; Reutlinger 1970) is that they provide "exact" solutions. We suggest however that, given the necessary simplifying assumptions regarding correlation between variables and regarding the shapes of the probability curves, this 'exactness' is illusory. Further, and more importantly, mathematical analysis of more complex problems is too time consuming to permit repeated testing of options - for example different assumptions concerning correlation or variations to project scale. Anyone interested in pursuing the mathematical approach is referred to Wagle (1967) or Zinn and Lesso (1977).

Monte Carlo Simulation

In this approach the economic evaluation model (eg for NPV or B/C ratio) is specified and probability distributions are developed for those costs and benefits which have a significant degree of uncertainty - point estimates may be made for others. The probability distribution of the dependent variable (NPV or B/C ratio) is then estimated by Monte Carlo processes. This involves random selection of a large number of sets of values, which conform to the respective distributions for the uncertain variables. A value of the dependent variable is then computed for each set of values for the uncertain variables - resulting in an estimated distribution for the dependent variable. Benjamin and Cornell (1970) give a simple illustration and a clear exposition of the technique.

The "true" distribution of the dependent variable would be established by taking all possible combinations of the independent variables. In practice depending on the complexity of the model, of the order of 500 to 1000 simulations will supply sufficiently accurate estimates of the mean and higher moments of the distribution - particularly given the inevitable simplifications and assumptions in the model building. Statistical tests such as the Kolmogorov-Smirnov test (Siegel, 1956) can be applied to assess the adequacy of the sample size, but in the case of fairly simple models it is sufficient merely to check the stability of the estimates over a range of sample sizes.[1]

AN APPLICATION OF PROBABILITY ANALYSIS USING MONTE CARLO SIMULATION: CASE STUDY OF WODONGA BYPASS

In 1976 the Bureau of Transport Economics evaluated a number of options for bypassing the towns of Albury and Wodonga (BTE, 1978). Amongst the options evaluated as a Wodonga Bypass. The bypass involved the construction of a four lane road linking the existing bridge across the Murray River and the Hume Highway south of Wodonga at a point outside of the existing built-up area. It was concluded that the Wodonga Bypass "with a benefit-cost ratio of 0.9, is not economically warranted for two or three years" (BTE 1978, p 63). The problems with deterministic approaches to benefit cost analysis discussed above are particularly important where the project is marginal[2] and there exists disagreement about the size of variables incorporated in the analysis. The Wodonga Bypass was such a case. For example, Victorian Country Roads Board did not agree with the conclusion that the economic warrant for the bypass was weak. "The Board's criticism is based on a disagreement over the forecast traffic on the bypass" (BTE 1978, p 75). The probabilistic approach can be particularly useful in these cases by providing information on the likelihood of the project being warranted given a range of values for uncertain variables.

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1 For a discussion of some of the often quoted disadvantages of the simulation approach see Singhvi (1980, pp 18-19).

2 With the present constraints on capital a benefit cost ratio of about 1.7 for a rural arterial road project would be marginal.

The Simulation Model

A copy of the computer program used in this exercise is available from the authors. In the most general terms the problem is to evaluate the net present value of the project where that is a function of a number of variables, for example average vehicle speed after the road improvement and traffic growth rates, and where there are probability distributions associated with some or all of these variables. The net present value will therefore also have associated with it some probability distribution. The objective of the simulation is to estimate that distribution.

For each of those variables that are uncertain the analyst must choose a type of probability distribution. Presently the program gives the analyst the choice of specifying a uniform, poisson or beta distribution. If a beta distribution is chosen the analyst must provide estimates of the lowest, most likely (modal) and highest value of the uncertain variable. The usual PERT assumption that the variance equals a sixth of the specified range is made (Hillier and Lieberman 1967, pp 208-234). For the poisson distribution the user must specify the average value the uncertain variable will take. In the case of the uniform distribution the analyst must estimate the lowest and highest value of the variable.

Given these estimates, a distribution is generated for each uncertain variable. Random values which conform to these distributions are then drawn and together with the certain variables are incorporated in the calculation of the net present value of the project. Random values are drawn and the net present value calculated one thousand times.

Choice of Probability Distribution

At present the program provides a relatively limited choice of distributions. A wide variety of other distributions could be incorporated including for example the normal, triangular and step rectangular distributions. A number of distributions are listed and discussed by Pouliquen (1970) and Cassidy and Gates (1977) in the context of simulation and by Zinn, et al (1977) in the context of a mathematical formulation approach.

The choice of distribution should be influenced by a comparison of the available information and the information required to specify the distribution, and by the source or nature of the uncertainity.

Pouliquen prefers the step rectangular distribution but this requires the analyst to provide information about the likelihood of a number of alternative values of the uncertain variable. Other distributions require the analyst to provide less information. For example, the beta and triangular distributions require the analyst to estimate the most likely value for the uncertain variable but not necessarily how likely that value is. Cassidy and Gates prefer the triangular rather than the beta because of its relative simplicity. Pouliquen (p 59) also argues that the triangular compensates for "the fact that one is tempted to assign to a value close to the extreme of a range of lower probability than to a value close to the best estimate". Tversky and Kahneman label this tendency the availability heuristic (1974). However this is not a general argument for choosing the triangular distribution. The extent to which those providing information from which the distribution is generated provide information biased through availability will differ across variables and situations.

The possible sources and nature of uncertainty were also discussed earlier in this paper. Choosing a distribution that has characteristics at odds with the source or nature of the uncertainty associated with a variable may bias results. Thus a normal distribution may be a reasonable representation of the uncertainty associated with an econometric estimate of a demand function but is unlikely to represent closely the uncertainty associated with the opening or continuation of a mining operation involving the use of a particular transport facility. Use of the uniform distribution is sometimes criticised because it seems intuitively wrong to delineate the boundaries of a range with certainty when the value the variable will take within the range is so uncertain. However, regulations and physical characteristics can often constrain the range of some highly uncertain variables.

The uncertain variables and the associated distributions and estimates of ranges and modes used in analysing the Wodonga Bypass are summarised in Table 5.

TABLE 5: WODONGA BYPASS SIMULATION
ASSUMPTIONS CONCERNING VARIABLES

Variable	Distribution	Lowest	Most Likely	Highest
Real growth in vehicle benefits (% pa)	Beta	0.0	2.0	2.5
Average speed for trucks on bypass (kph)	Beta	50	70	85
Average speed for cars on bypass (kph)	Beta	70	90	100
Total constructions cost ($m)	Beta	8	10.5	13
Percentage of terminating traffic using bypass (%)	Beta	32	42	52
Discount rate (%)	Beta	7	10	13
Value of car time ($)	Beta	0.0	3.8	4.5
Value of truck time ($)	Beta	3.6	4.6	5.6
Cars in bypass traffic (%)	Beta	55	60	70

Treatment of Correlation

A basic assumption of the simulation approach is that each of the uncertain variables are independent. For those uncertain variables that significantly effect the net present value and are correlated, choosing values that are not related will affect the accuracy with which the simulated distribution reflects the (unknown) true distribution. For example, when variables are drawn at random and independently they will often offset each other. If positively correlated, then one variable should amplify the effect of the other. Thus ignoring positive correlation by treating related variables as independent will lead to underestimation of the variance in the distribution of the net present value.

Correlation can be accounted for by aggregating variables, ie including less detail in the model of net present value or by specifying variables and relationships in more detail.(1) In aggregating the analysis the product or sum of correlated variables is included in the analysis as the independent uncertain variable rather than the correlated variables. Values of the product or sum variable are then drawn and used in the simulation.

However, aggregation may reduce the accuracy of the simulation because of difficulty in estimating the range the aggregated variable is likely to take. This difficulty reflects the lack of knowledge about the true relationship between the more specific but correlated variables. If the relationship between the correlated variables is known or can be estimated then this should be explicitly incorporated in the analysis. In this case the values for the truely independent variable (and perhaps the parameters of the relationship) are drawn randomly and used in the simulation.

In the analysis of the Wodonga Bypass a number of correlations were identified. In the model vehicle operating cost and time savings are both a function of vehicle speed, maintenance cost is a function of traffic and traffic growth rates are a function of the real growth in vehicle benefits.(2)

Results of Wodonga Bypass Simulation

Using the evaluation subroutine included in the simulation program and with all variables fixed at their 'most likely' level (the deterministic approach) the project has a net present value of approximately $0.2 million. This gives a benefit cost ratio of about 1.0.(3) Using

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1 These and other techniques are discussed in greater detail in Rentlinger (1970), Pouliquen (1970) and Harrison and Cassidy (1977).

2 The vehicle operating cost functions and maintenance cost functions are taken from Commonwealth Bureau of Roads (1977) and Both and Bayley (1976) respectively. The growth rate function was estimated for illustrative purposes.

3 The results generated by this model differ marginally from the results published by the Bureau of Transport Economics (1978) because of simplifications introduced into the evaluation subroutine used in the simulation.

the distributions and estimates shown in Iable 5 the simulation approach produced a distribution of net present values with a mean of -$0.9 million and a standard deviation of $1.6 million. The difference between the result of the deterministic approach and the expected value of the project given by the simulation is over $1.0 million (10 per cent of the discounted cost of the project). This demonstrates the inaccuracy of the deterministic approach in estimating the expected value of the project when the distributions of the value of the uncertain variables is skewed. The upper and lower bounds of the range of outcomes at the 95 per cent confidence limit were $2.7 million and -$3.7 million.

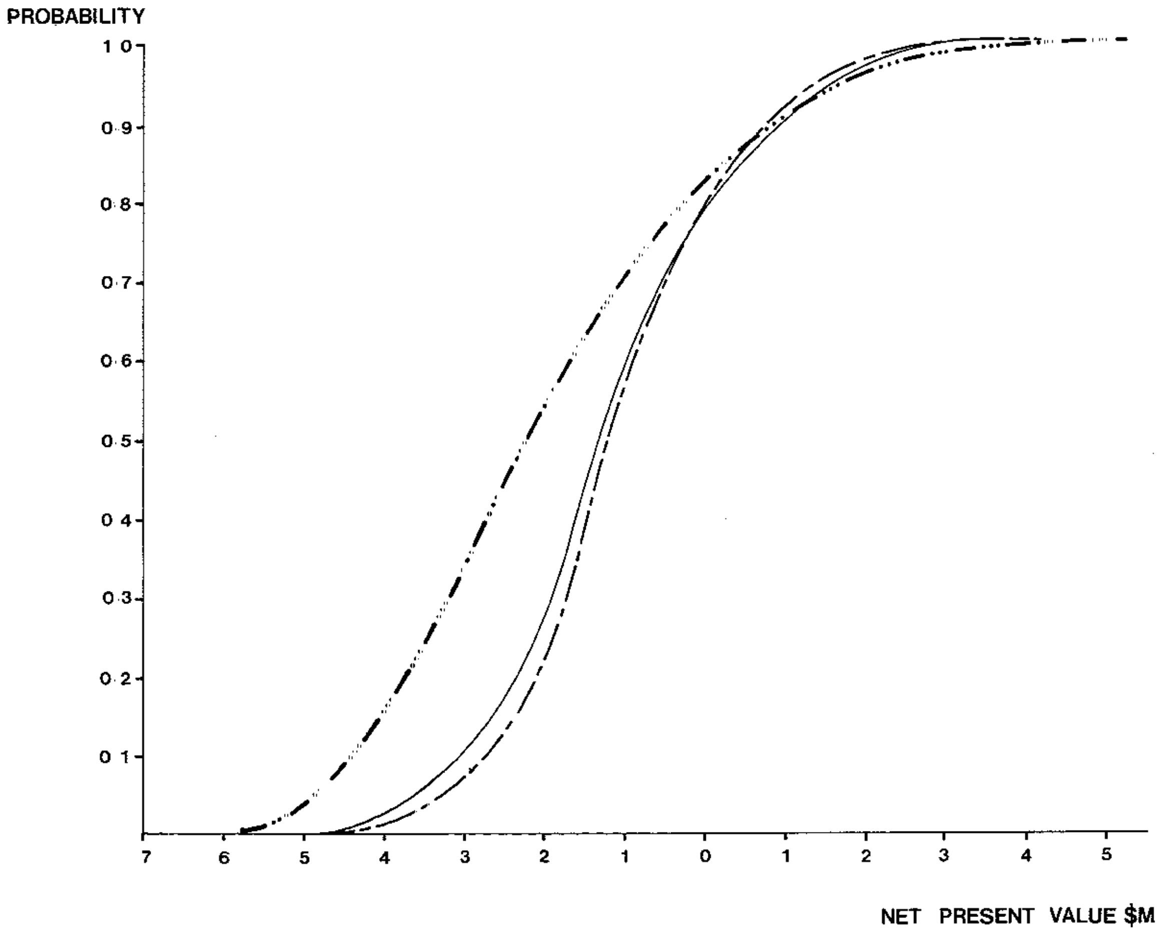
Figure 2 further demonstrates the inadequacy of the deterministic approach. The cumulative probability distribution derived from the simulation indicates that given the distributions on the uncertain variables there is about an 80 per cent chance of the value of the project being less than or equal to the value given by the deterministic approach. Thus, given the uncertainty associated with many of the variables in the analysis, it is unlikely that the project would provide the outcome estimated using the deterministic approach. However it should also be noted that there is over a 65 per cent chance of the true value of the project being less than or equal to the expected value generated by the Monte Carlo model.

Sensitivity Analysis

Sensitivity analysis can be used to complement the simulation approach in a manner similar to the way it is used in a deterministic approach. The analysis involves testing the sensitivity of the results to the characteristics of the distribution of the value of each of the uncertain variables. Although providing a guide to where greatest effort in estimation should be directed it still cannot indicate the likelihood of the effects of a combination of 'errors' occurring in the simulation.

Figure 2 also shows the results of a test of Pouliquen's result that correlation is "... a much more serious problem than the choice of the probability distributions" (Pouliquen 1970, p 61). Two extra simulations were run. In the first of these an element of correlation was ignored by allowing a real growth in vehicle benefits and the growth in traffic to vary independently. The range and most likely values for vehicle growth were set at those that would be given by the relationship between traffic growth and real growth in vehicle benefits and by the range and most likely value of real growth in vehicle benefits. A beta distribution was specified for vehicle growth rates. In the second simulation a uniform distribution was specified for real growth in vehicle benefits and the correlation between this and traffic growth was left intact.

As indicated in Figure 2 changing the specified distribution on real growth in vehicle benefits had far more effect on the cumulative probability distribution than removing the correlation between that variable and traffic growth. The uniform distribution also reduced the expected value of the project by $0.9 million while removing the correlation left the expected value of the project unchanged. This is evidence that Pouliquen's results are at least not correct as a general rule. It demonstrates that the choice of probability distribution can be a more important problem than correlation.



Result of Simulation

'Without Correlation' Sensitivity Test of Results of Simulation

'With Uniform Distribution' Sensitivity Test of Results of Simulation

FIGURE 2: CUMULATIVE DISTRIBUTION FUNCTIONS

As is to be expected, the change from a beta to a uniform distribution for real growth in vehicle benefits has its greatest impact on the tails of the distribution. Thus in this case the probability of the value of the project being less than that given by the deterministic approach and being less than the expected value given by the simulation is relatively insensitive to the form of the distribution on this variable.

CONCLUSION

This paper has outlined the weaknesses of the traditional deterministic approach and the strength of the probabilistic techniques in applied benefit cost analysis. We have demonstrated these points in relation to an actual case study and shown that it is comparatively easy to apply this technique. We contend that probabilistic benefit cost analysis should be more widely applied in transportation project appraisal.

APPENDIX 1: SUFFICIENCY OF EXPECTED VALUE

Although the question is worthy of a paper in its own right, we digress at this point to consider very briefly why higher moments of the distribution of present value and not just the expected value are relevant to public decision making. Dasgupta and Pearce (1972), for example, argue that only expected value is relevant.

Samuelson (1964), Vickrey (1964) and Arrow (1964) argue that the variance of the returns to government investment, and hence the attendant risk or uncertainty, is reduced to insignificance as a result of "risk pooling" - that is, the losses resulting from projects that fall short of expectations will be balanced by the gain from projects that do better than expected. This argument rests on the statistical law of large numbers which implies that, given a very large number of diverse projects whose returns are independent of each other, the variance approaches zero. The risk pooling argument must be distinguished from the risk spreading argument developed by Arrow and Lind (1970). Arrow and Lind show that, as the net returns to an investment are shared by an increasingly large number of individuals, the individual risk premium, and the aggregate of all such premiums goes to zero. To the extent then that the benefits of publically funded projects accrue to the population at large, the individual cost of risk is minimal and the expected value is the relevant measure of the worth of a project

Hirshleifer (1965 and 1966) and particularly Hirschleifer and Shapiro (1970) effectively dispose of the risk pooling argument, but the risk spreading thesis of Arrow and Lind still has applicability in certain situations. There are however reasons why risk spreading also is irrelevant in many public sector evaluations, and hence why it is desirable to take second or higher moments of a probability distribution into account in public sector decision making.

First, and most critical, much public investment is for purpose of production of public goods in the Samuelsonian sense. Because public goods are indivisible and property rights in them cannot be established by individuals, any risk attaching to them is correspondingly indivisible (Fisher 1973; Fisher and Krutilla 1974; Arrow and Fisher 1974; Shepsel 1977). This limitation on the Arrow and Lind thesis is particularly relevant where environmental externalities are significant.

Secondly, efficient risk spreading requires that the covariance of a given marginal project's income with all other private and public projects, and hence all forms of income, should be close to zero (Mayshar 1977). The project's return should not be highly correlated, for example, with the level of national income. Sandmo (1972) and Bailey and Jensen (1972) among others suggest that a key assumption of Arrow and Lind, that a given project's returns are independent of aggregate income because stabilisation policy is always successful in maintaining full employment levels of income, is at odds with the real world, and is thus untenable.

Thirdly, the Arrow and Lind assumptions are not compatible with a situation where there is consistent bias introduced into evaluation and as discussed above there is some evidence of this.

Finally, we note that even where the expected value is the only relevant criteria for decision making, use of probabalistic evaluation techniques will usually lead to better estimates of the expected value, than will the traditional deterministic approaches. This is demonstrated in the case study in this paper.

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