

A MODIFIED ALGORITHM FOR DYNAMIC ADVISORY SPEED SIGNS

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ABSTRACT: *Advisory speed signs are designed to facilitate the formation of platoons and promote the flow of traffic in a 'green wave'. The speeds are computed to create a safe delay between the start of the green phase and the arrival of the platoon at speed. Observation of the performance of an earlier algorithm in an on-road trial suggested that drivers tended to arrive at the signals too early when very low speeds were advised. This paper presents a modification to the original algorithm design to correct for this by adjusting for the time and distance required by vehicles to slow to the advised speed.*

INTRODUCTION

Dynamic advisory speed signs (von Stein, 1959, and Morrison *et al.*, 1962) have hitherto been used to solve traffic problems at specific locations (tunnels/freeway exits) with the reduction in travel time receiving less emphasis. The concept behind advisory speed signs is the promotion of the flow of traffic in a 'green wave'. In its most readily applied form the advisory signs are an addition to a fixed time traffic control system. The speeds displayed are calculated to facilitate the formation of platoons, and to move these platoons along successive links within the green wave. The signs give the driver the information necessary to select a suitable speed, and operate as a form of feedback control by supplying updated speed advice as the vehicle moves down the road. Typically the advisory speed signs are located just downstream of the previous signalized intersection and within long links at 1/2 km intervals. The displayed speeds are computed to create a safe delay between the start of the green phase and the arrival of the platoon at speed, and are constrained to fall within a practical operating range.

The most obvious benefit to the driver for complying with the system is a reduction in the number of stops. Discrete simulation (Doughty and Trayford, 1982) showed that in a variety of situations stops can be reduced to near zero given full compliance by drivers. A further advantage is the reduction in fuel consumed.

Other advantages arising from the system are likely to benefit the community as much as the individual driver. Near uniform speeds reduce noise, hydrocarbon emissions, lane changing and produce a lower speed differential between adjacent vehicles. The simulated reduction in lane changing has been documented by Doughty and Trayford (1982) and as a consequence, a lower incidence of accidents could be expected.

This paper takes the original algorithm proposed by Doughty *et al.* (unpublished data) and develops a modification that on-road trials suggest might be appropriate in certain circumstances.

THE ORIGINAL MODEL

The simplest approach to the problem of calculating an advisory speed would be to employ the minimum of the speed limit, and the speed, which if adopted, would cause the driver to cross the stop line as the signals turned green. However, some drivers may be reluctant to approach a red signal at a speed which would prevent them from stopping in time should the system malfunction. Consequently attempts to place the leading vehicles on the stop line when the signals turn green is liable to suffer from disturbances produced by late braking as occasional drivers slow down to what they consider is a safe speed just in case the signals do not change. Since the basic goal of dynamic advisory speed signs is to smooth the flow of traffic, displaying a speed which will place the leading drivers in a platoon in a position where some may feel compelled to brake is self defeating.

The approach adopted by Doughty *et al.* (unpublished data) was for the signs to advise a speed which would place the leading vehicles some distance back from the stop line when the signals were due to change. The combination of distance and speed being chosen to maximize the approach speed of the driver (within limits) while keeping the likelihood of drivers braking to a minimum. That is, the signals should

turn green at the point where the drivers would otherwise make the decision to start braking.

Let

- X be the location of the stop line,
 x be the location of the advisory sign ($x < X$),
 T be the time when the traffic signals are due to turn green,
 t be the current time,
 β (< 0) be a braking rate typical of drivers making a late decision to stop, and
 $s_1(t, x)$ be the unconstrained speed produced by these conditions.

If t is less than T , a vehicle which complies with the advised speed will travel a distance $(T-t) \cdot s_1(t, x)$ beyond the sign before the signals turn green. If the signals fail to turn green and the driver brakes, the vehicle will travel a further distance of $-s_1(t, x)^2 / 2\beta$ before stopping. For safety, the vehicle should not have reached the stop line (Figure 1), hence

$$x + (T-t) \cdot s_1(t, x) - s_1(t, x)^2 / 2\beta \leq X \quad (1)$$

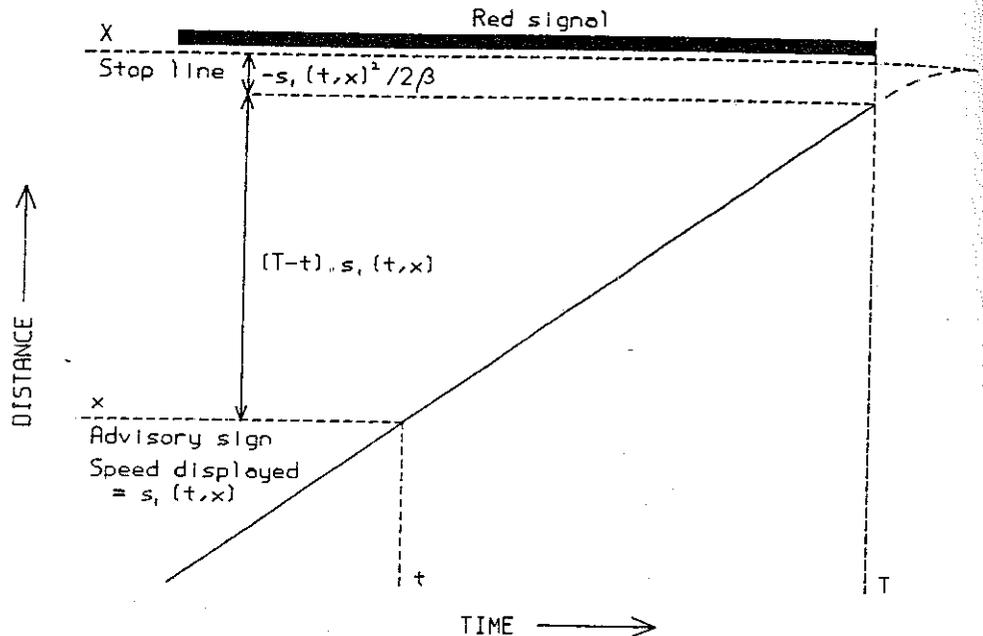


Figure 1 Time-distance diagram illustrating the derivation of the original advisory speed algorithm.

Rearranging and solving yields

$$s_1(t, x) \leq \beta(T-t) + [\beta^2(T-t)^2 - 2\beta(X-x)]^{1/2} \quad (t < T) \quad (2)$$

This expression is subject to a number of constraints. Speeds are bounded above, where one of the possible bounds is the speed limit; and below, to prevent too great a speed differential between those

drivers who choose to comply with the sign and those who do not. Displaying the lower limit in the case of speeds which would otherwise fall below it, will result in drivers who comply with the system being forced to slow or stop at the intersection, reducing their credence in the system. Consequently, the sign displays an "X" or other symbol in these circumstances to indicate that the speed is undefined. Thus

$$s(t,x) = \begin{cases} \text{undefined} & s_1(t,x) < S_L \\ s_1(t,x) & S_L \leq s_1(t,x) < S_U \\ S_U & S_U \leq s_1(t,x) \end{cases} \quad (3)$$

where,
 S_U is the upper limit to the advisory speeds,
 S_L is the lower limit to the advisory speeds, and
 $s(t,x)$ is the speed actually displayed by the advisory sign.

Provision is also made for changing the green phase for which the advisory speeds are calculated when a vehicle travelling at the maximum speed the sign will advise cannot reach the intersection before the signals turn red.

DEPARTURES FROM THE ORIGINAL MODEL

The performance of the algorithm was evaluated in an on-road trial using an instrumented car equipped with a micro-computer programmed to calculate the advisory speed on the basis of the vehicle's location and the known cycle timings (Trayford *et al.*, unpublished data). The experiments involved a single signalized intersection, and observation suggested that drivers were tending to arrive at the signals too early when very low speeds were advised. This could have been caused by a number of different factors, the two leading contenders being: that drivers were reluctant to travel at low speeds when the vehicles around them were travelling much faster; or that the drivers who were travelling near the mean free speed when advised of an appropriate speed took an appreciable time to slow down to the lower speeds and consequently arrived too early. The first possibility is less likely to be a problem with advisory signs positioned beside the road when all drivers (not just one) would be advised of a suitable speed. The second possibility is one for which an algorithmic solution exists and is the subject of this paper.

The model represented in equation (2) was based on the assumption that the vehicle cruised at a constant speed $s_1(t,x)$ from the moment it passed the advisory sign. However, if the vehicle is travelling faster than the posted speed the driver will first slow down, probably at a relatively gentle rate, before cruising at the advisory speed. The time taken to slow down to the advisory speed and the distance covered while doing so, will bring the vehicle closer to the stop line than was assumed desirable in the derivation of the algorithm. As a consequence the driver of the vehicle may undertake further braking as he approaches the intersection, possibly even having to stop completely as shown in Figure 2.

The magnitude of the discrepancy produced by assuming a constant cruising speed can be estimated relatively easily. If the driver is initially travelling at σ and slows down to the advised speed, $s_1(t,x)$, at some rate, β^* (<0), his vehicle will take a time, $(s_1(t,x)-\sigma)/\beta^*$, to reach the advised speed, during which it will have travelled a distance, $(s_1(t,x)^2-\sigma^2)/2\beta^*$. Had the driver been cruising at the advised speed

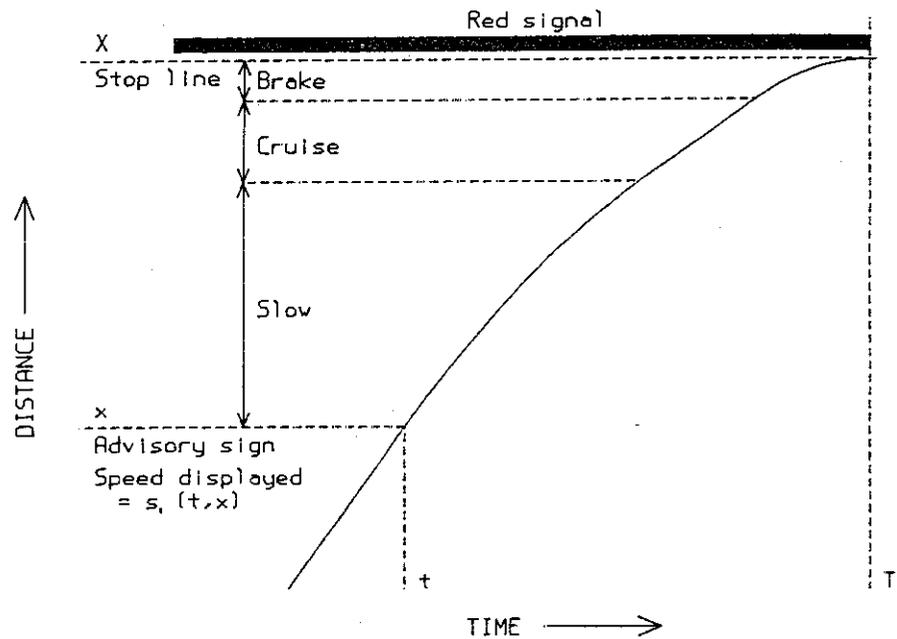


Figure 2 Time-distance diagram illustrating the effect of the initial speed of a vehicle on the time of its arrival at the traffic signals.

all the time, he would have taken a time $(s_1(t,x)^2 - \sigma^2) / 2\beta^* s_1(t,x)$ to cover this distance. As a consequence, by the time the driver has slowed to the advised speed, he will be ΔT too early on his arrival at any given point along the road, where ΔT is given by

$$\Delta T \approx (s_1(t,x)^2 - \sigma^2) / 2\beta^* s_1(t,x) - (s_1(t,x) - \sigma) / \beta^*$$

or after some manipulation,

$$\Delta T = (s_1(t,x) - \sigma)^2 / 2\beta^* s_1(t,x) \quad (4)$$

The nature of the dependence on the advisory and initial speeds of the error thus induced is illustrated in Figure 3, where the time by which the vehicle arrives too early is plotted against $s_1(t,x)$ for various values of σ . From Figure 3 it can be seen that this error is very large for low advisory speeds, but drops rapidly as the advisory speed increases. The rate of decline from the initial peak is also significantly affected by the initial speed of the vehicle. The value of β^* was held at -1.0 m/s/s in all cases illustrated as its only effect is to alter the vertical scale, without changing the shape.

THE MODIFIED MODEL

A modification to the original algorithm to overcome this problem is obtained by supposing that the vehicle is travelling at a speed, σ , when the driver first observes the advisory speed sign. (It is assumed that σ is greater than the advised speed). The driver is unable to reduce

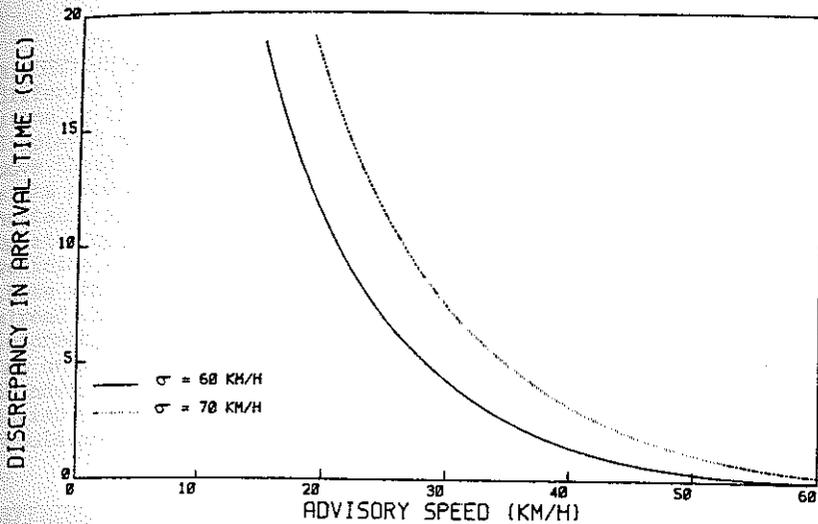


Figure 3 Plot of the time by which a vehicle arrives early at the traffic signals against the advisory speed for initial speeds of 60 and 70 kilometres per hour.

his speed instantaneously, but slows to the advisory speed, $s_2(t,x)$, at some rate, β^* . Slowing down from σ to $s_2(t,x)$, at β^* will take a time, $(s_2(t,x)-\sigma)/\beta^*$, while the vehicle travels a distance $(s_2(t,x)^2-\sigma^2)/2\beta^*$. This leaves a time interval of $T-t-(s_2(t,x)-\sigma)/\beta^*$ before the signal turns green. Cruising at the advisory speed during this time will enable the vehicle to cover a distance of $[T-t-(s_2(t,x)-\sigma)/\beta^*] \cdot s_2(t,x)$. If the signals fail to turn green and the driver brakes, the vehicle will travel a further distance of $-s_2(t,x)^2/2\beta$ before stopping. For safety, the vehicle should not have reached the stop line (Figure 4), hence

$$x - (\sigma^2 - s_2(t,x)^2)/2\beta^* + [T-t + (\sigma - s_2(t,x))/\beta^*] \cdot s_2(t,x) - s_2(t,x)^2/2\beta \leq X \quad (5)$$

Rearranging and solving yields

$$s_2(t,x) \leq [-(T-t + \sigma/\beta^*) + [(T-t + \sigma/\beta^*)^2 + 2B(X - x + \sigma^2/2\beta^*)]^{1/2}]/B \quad (t < T) \quad (6)$$

where,
 $B = -(1/\beta + 1/\beta^*)$

This expression is now subjected to the same constraints with respect to upper and lower bounds that applied to the original, and $s_2(t,x)$ can be substituted for $s_1(t,x)$ in equation (3).

DISCUSSION

The modified algorithm described in this paper was designed to correct the problem of drivers arriving at the traffic signals too early because they took an appreciable time to slow down from their initial

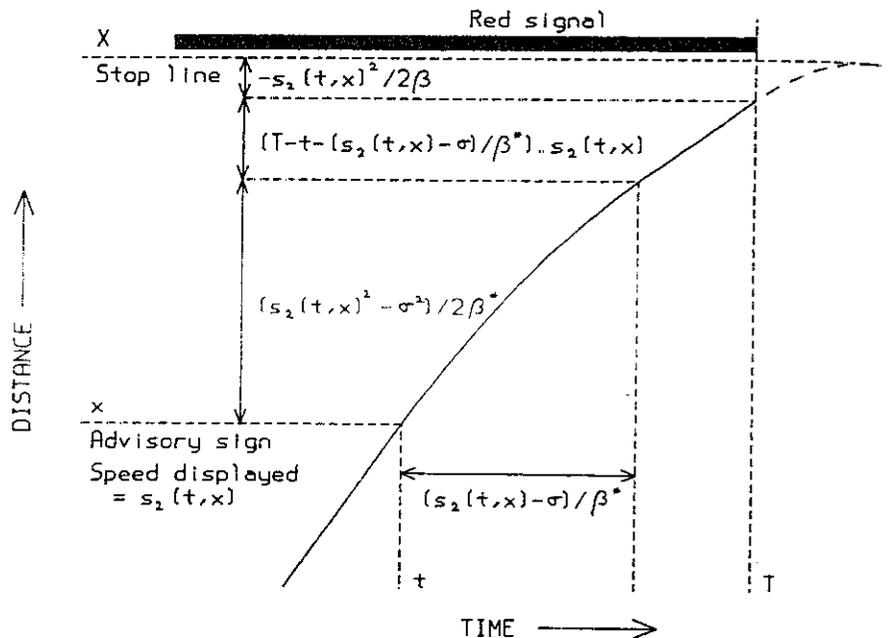


Figure 4 Time-distance diagram illustrating the derivation of the modified advisory speed algorithm.

speed to the advisory speed. No attempt was made to calibrate the model because of a lack of sufficient data.

Because of the two additional parameters involved, β^* and σ , the modified algorithm will require more effort to calibrate than the original. Further, the time that the vehicle arrives at the traffic signals is very sensitive to the initial speed of the vehicle, and the rate at which the driver slows to the advised speed. Both these values are likely to vary substantially between vehicles in a traffic stream and have a corresponding effect on the performance of the algorithm. Thus, it is worth considering the magnitude of the difference in the respective advisory speeds and which algorithm is more suitable in particular circumstances.

Reference to Figure 3 shows that when vehicles have to reduce their speed by less than 20 km/h, the difference between the arrival times produced by the two algorithms is less than one second. This represents the difference between an average speed of 43 km/h and one of 45 km/h over 300 metres, probably the limit to which one could expect drivers to perform on a routine basis in traffic. Thus, the modified algorithm is likely to be of most use in those situations where the drivers may have to reduce their speed by more than 20 km/h if they wish to comply with the advisory sign. This is most likely to occur when vehicles travelling at their mean free speed first enter an advisory speed zone. In the trial described by Trayford *et al.* (personal communication) where the discrepancy with the original algorithm was noted, the mean free speed of the traffic was in excess of 70 km/h and vehicles were approaching a single intersection.

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Conversely, the modified algorithm will be of lesser use in situations where the advisory speed is not likely to be markedly different from the initial speed. One of the aims of advisory speed signs is to promote the flow of traffic in a 'green wave'. The speeds displayed are calculated to facilitate the formation of platoons, and to move these platoons along successive links within the green wave. The signs provide the driver with the information necessary to select a suitable speed, and operate as a form of feedback control by supplying updated speed advice as the vehicle moves down the road. Thus, once vehicles have passed the first advisory sign in a coordinated system, those vehicles whose drivers comply with the advisory signs should already be travelling at approximately the correct speed and the information from the signs will be used to adjust speeds rather than set new ones. Consequently it can reasonably be expected that once within an advisory speed zone the difference between the initial speed and the advisory speed will be relatively small and the original algorithm should prove satisfactory.

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