

COMPARISONS OF TRANSIT STOP SPACING POLICIES

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ABSTRACT

Given a number of stops and line length, where should the stops be set for a transit route? This paper uses mathematical derivations to compare three transit performance indicators (ridership, transit round trip travel time and passenger walking distance) under different transit stop spacing policies. An empirical study is also presented to support the results of the theoretical derivations. Four policy implications for transit stop spacing are suggested in the paper: (1) transit operators would prefer equal demand spacing rather than equal distance spacing if the underlying travel demand function is convex; (2) transit round trip travel and operator cost under equal distance spacing policy should be lower under equal demand spacing; (3) total passenger walking distance under equal distance spacing and equal demand spacing should be the same; and (4) if the travel demand is equally distributed then total walking distance under equal distance spacing (or equal demand spacing) can reach minimum total walking distance.

1. INTRODUCTION

The purpose of this paper is to clarify some of the arguments about transit stop spacing policies. Given a number of stops and line length, four transit spacing policies related to travel demand have been proposed in the literature: (1) uniform equal spacing: stops are equally spaced along the transit line irrespective of travel demand (Holroyd, 1965; Lesley, 1976); (2) inverse demand spacing: spacings of transit stops should be inversely related to the ratio of passenger origins and destinations to volume of passengers traveling through an area (Vuchic, 1981); (3) inverse square root demand spacing: spacings of transit stops in a linear transit route should be inversely proportional to the square root of the number of passenger boarding and alighting¹. (Vaughan and Cousins, 1977; Webster and Bly, 1979; Kush and Perl, 1988; Wirasinghe and Ghoneim, 1981); and (4) equal demand spacing: the number of boarding and alighting passengers is equal for each stop.

With mathematical derivation, we investigate some of the properties of spacing policies and discuss general characteristics. Given the number of stops (determined by the budget), three interesting questions related to transit spacing policies are the following :

1. Which spacing policy will attract more transit passengers and operator revenue?
2. Which spacing policy will achieve the minimum transit round trip travel time and operating cost?
3. From viewpoint of passengers, which spacing policy will minimise total access time to transit stops?

Sections 2 to 4 discuss these three issues respectively. A empirical study is presented in section 5. Finally, some policy implications are made in the last section.

2. TRANSIT TRAVEL DEMAND (OPERATOR REVENUE)

Transit travel demand (modal split) is a function of travel times and travel costs on all transport modes. There are two reasons why we could assume that all variables are constant, with the exception of walking time, in the travel demand function for comparing travel demand under different spacing policies.

¹'square root' on a linear route should be replaced by 'cube root' in a two-dimensional city (Vaughan, 1986)

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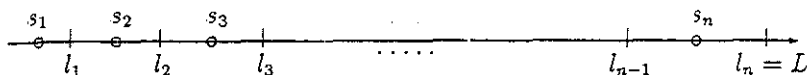


Figure 1: Transit Stops Along the Route

1. Auto travel time, auto travel cost and transit fare are independent of spacing policy.
2. In designing bus stop locations, access time (walking time to bus stop) is the only significant factor influencing transit travel time among the four components of passenger travel time.

Thus the travel demand model could be simplified as a function of walking distance for comparing travel demand under different spacing policies.

$$D_i = f(w) \quad (1)$$

where f is travel demand function, and D_i is travel demand (including auto and transit) at stop i and w is walking distance.

Consider a given transit line with line length L as shown in Figure 1, for which $\{s_i, i = 1, 2, \dots, n\}$ are distances from the first stop to stops $\{S_i, i = 1, 2, \dots, n\}$ and $\{l_i, i = 1, 2, \dots, n\}$ are the hinterland boundaries of the stops, e.g. trips started or finished between l_{i-1} and l_i will use stop S_i . Under equal spacing policy, total transit passenger demand is

$$\sum_{i=1}^n D_i f\left(\frac{L}{n}\right) = D_T f\left(\frac{L}{n}\right) \quad (2)$$

where D_T is total travel demand. L is transit line length. On the other hand, total transit travel demand under equal demand policy is

$$\sum_{i=1}^n \frac{D_T}{n} f(l_i - l_{i-1}) = \frac{D_T}{n} \sum_{i=1}^n f(l_i - l_{i-1}) \quad (3)$$

According to Jensen's Inequality² (see Mitrinovic, 1964) we obtain

$$\begin{aligned}
 f\left(\frac{\sum}{n}\right) &< \frac{1}{n} \sum_{i=1}^n f(l_i - l_{i-1}) && \text{if } f(w) \text{ is strictly convex.} \\
 &= \frac{1}{n} \sum_{i=1}^n f(l_i - l_{i-1}) && \text{if } f(w) \text{ is linear.} \\
 &> \frac{1}{n} \sum_{i=1}^n f(l_i - l_{i-1}) && \text{if } f(w) \text{ is strictly concave.}
 \end{aligned}
 \tag{4}$$

In other words, equal demand spacing would attract more passengers if the $f(w)$ were strictly convex and less if it were concave. Transit operators would prefer equal demand spacing if the $f(w)$ were convex because it would attract more passengers and hence increase revenue. On the other hand, equal spacing would be preferred by transit operators if $f(w)$ were concave.

3. TRANSIT ROUND TRIP TRAVEL TIME (OPERATING COST)

Three components of a transit travel time (from the first stop to the last) are included: passenger boarding and alighting time, constant-speed travel time, and the additional acceleration and deceleration time (lost time for stopping). Among them, the transit additional acceleration and deceleration time is the only factor affected by the different transit stop spacing policies.

Suppose the spacings are long enough that transit vehicles can accelerate to maximum speed. The expected value of lost time from the first stop to the last is

$$\sum_{i=1}^n \frac{V_m}{2} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) [1 - \exp(-D_{bi})] \tag{5}$$

(see Ling and Taylor, 1988), where D_{bi} is the transit demand at i , V_m is the maximum transit speed, r_a and r_b are transit acceleration and deceleration rate. For equal demand spacing policy, total expected lost time from the first stop to the last is

$$\frac{V_m}{2} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) \sum_{i=1}^n [1 - \exp\left(-\frac{D_{bi}}{n}\right)] = \frac{V_m}{2} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) [n - n \exp\left(\frac{1}{n} \sum_{i=1}^n -D_{bi}\right)] \tag{6}$$

On the other hand, for equal spacing policy,

$$\frac{V_m}{2} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) \sum_{i=1}^n [1 - \exp(-D_{bi})] = \frac{V_m}{2} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) [n - \sum_{i=1}^n \exp(-D_{bi})] \tag{7}$$

²For every convex function $f(x)$,

$$f\left(\frac{1}{n} \sum_{i=1}^n a_i\right) \leq \frac{1}{n} \sum_{i=1}^n f(a_i)$$

Since the negative exponential function is convex, we could obtain (8) according to Jensen's Inequality.

$$n \exp\left(\frac{1}{n} \sum_{i=1}^n -D_{bi}\right) \leq \sum_{i=1}^n \exp(-D_{bi}) \quad (8)$$

This expression indicates that expected lost time under equal demand spacing policy would be greater than that under equal spacing policy. The shorter transit round trip travel time allows for small fleet size. In other words, transit round trips travel time as well as operating cost under equal demand spacing policy would be greater than that under equal spacing policy.

4. PASSENGER WALKING DISTANCE

Let us further specialise to the case where all transit passengers access transit stops by walking, the average walking distance to the nearest transit stop is approximately one-fourth of the distance between two stops. Therefore the total walking distance is given by:

$$W = \frac{1}{4} \sum_{i=1}^n (l_i - l_{i-1}) [g(l_i) - g(l_{i-1})] \quad (9)$$

where W is the total walking distance and g is the passenger demand as a continuous function of the distance from the first stop.

Under equal spacing policy,

$$W = \frac{1}{4} \sum_{i=1}^n \frac{L}{n} [g(l_i) - g(l_{i-1})] = \frac{LD_b}{4n} \quad (10)$$

where D_b is the total transit travel demand.

On the other hand, walking distance under equal demand policy is

$$W = \frac{1}{4} \sum_{i=1}^n (l_i - l_{i-1}) \frac{D_b}{n} = \frac{LD_b}{4n} \quad (11)$$

The results of equations (10) and (11) show that total walking distance under equal spacing and equal demand spacing would be equal.

Suppose the travel demand is equally distributed, i.e. $g(x) = k$, where k is a constant. Then,

$$W = \frac{k}{4} \sum_{i=1}^n (l_i - l_{i-1})^2 \quad (12)$$

Under equal spacing,

$$W = \frac{k}{4} n \left[\frac{1}{n} \sum_{i=1}^n (l_i - l_{i-1}) \right]^2 \quad (13)$$

According to Jensen's Inequality we obtain

$$n \left[\frac{1}{n} \sum_{i=1}^n (l_i - l_{i-1}) \right]^2 \leq \sum_{i=1}^n (l_i - l_{i-1})^2 \quad (14)$$

In other words, if the travel demand is equally distributed then equal spacing (as well as equal demand spacing) can achieve minimum total walking distance. In this case, passengers would prefer equal spacing policy (or equal demand spacing policy). Vaughan and Cousins (1977) made the same conclusion by using a continuous model which described the trip origins and destinations along the bus route as a continuous function of distance from the first stop. The model was solved numerically to obtain the optimum bus stop spacing.

For the general case of minimising total walking distance, we take derivatives of equation (9) with respect to $l_i, i = 1, 2, \dots, n-1$, set them equal to zero, and solve simultaneously,

$$\begin{cases} \frac{\partial W}{\partial l_1} = \frac{1}{4} [2g(l_1) - g(l_0) - g(l_2) + (2l_1 - l_0 - l_2)g'(l_1)] = 0 \\ \frac{\partial W}{\partial l_2} = \frac{1}{4} [2g(l_2) - g(l_1) - g(l_3) + (2l_2 - l_1 - l_3)g'(l_2)] = 0 \\ \vdots \\ \frac{\partial W}{\partial l_{n-1}} = \frac{1}{4} [2g(l_{n-1}) - g(l_{n-2}) - g(l_n) + (2l_{n-1} - l_{n-2} - l_n)g'(l_{n-1})] = 0 \end{cases} \quad (15)$$

This yields a set of $n-1$ nonlinear equations. Since l_n and l_0 are given, the equations contain $n-1$ unknown variables. Thus we can find a feasible solution $\{l_i, i = 1, 2, \dots, n-1\}$ such that the total walking distance is minimum. There are several computer packages (e.g. IMSL) for solving such a system of nonlinear equations. However, the set of equations above has a special structure. Each equation contains only three unknown variables with the exception of the first and the last equations which contain only two variables each. The generalised algorithm of solving nonlinear equations is inefficient, or perhaps unable to find the global optimum solution for a large number of equations. In this paper we develop a special method to solve those equations. It is derived in following steps:

1. assume an initial value of l_1 to solve l_2 through the first equation;
2. since l_1 and l_2 are determined, we could obtain l_3 by solving the second equation. Similarly, l_4, l_5, \dots, l_n could be obtained

3. l_n should be approximately equal to L . If the differences between l_n and L are too large, set new value of l_1 and repeat steps 1 and 2 until satisfactory convergence is reached

Compared with the IMSL subroutine, this method has proved more efficient (less computer CPU time) in various numerical examples, while the results are almost identical.

5. EMPIRICAL STUDY

We compare total walking distance and transit travel time under six spacing policies on the basis of an empirical study. The spacing policies are: existing system, minimum walking distance spacing, equal spacing, equal demand spacing, inverse demand spacing, and inverse square root demand spacing.

The bus route chosen for this study is the Melbourne Bus Route 700 which begins at Mordialloc, runs along Warrigal Road, and ends at Box Hill. It is 25.2 km long and contains 97 bus stops. The average bus stop spacing is approximately 260 metres. A bus trip O-D survey was carried out in 1985 by Denis Johnston and associates Pty Ltd under contract to the MTA. A total of 1811 single journeys were recorded for the route. 30.3 per cent of the trips occurred during the peak hour (4pm to 5pm). The calculations in the following are based on the peak hour data. Denis Johnston and Associates (1987) discussed the details of the data collection method and procedures.

The continuous cumulative function of boarding and alighting passengers, $g(x)$, is represented by a polynomial function of the distance from the first stop. It was estimated from the observed bus trip O-D survey data described above.

The hinterland boundaries of the transit stop locations, $\{l_i, i = 1, 2, \dots, n\}$, under different spacing policies are calculated as follows.

Equal Spacing

$$l_i = \frac{iL}{n} \quad i = 1, 2, \dots, n \quad (16)$$

Equal Demand Spacing

$$g(l_i) - g(l_{i-1}) = \frac{g(L)}{n} \quad i = 1, 2, \dots, n \quad (17)$$

Since l_0 is given, we can obtain $\{l_i, i = 1, 2, \dots, n\}$ by solving the set of equations

Inverse Demand Spacing

$$\begin{cases} (l_1 - l_0)[g(l_1) - g(l_0)] = (l_2 - l_1)[g(l_2) - g(l_1)] \\ (l_2 - l_1)[g(l_2) - g(l_1)] = (l_3 - l_2)[g(l_3) - g(l_2)] \\ \vdots \\ (l_{n-1} - l_{n-2})[g(l_{n-1}) - g(l_{n-2})] = (l_n - l_{n-1})[g(l_n) - g(l_{n-1})] \end{cases} \quad (18)$$

The method of solving minimum walking distance as mentioned as in section 4 can be applied to find the solutions for the set of equations above

Inverse Square Root Demand Spacing

$$\begin{cases} (l_1 - l_0)\sqrt{g(l_1) - g(l_0)} = (l_2 - l_1)\sqrt{g(l_2) - g(l_1)} \\ (l_2 - l_1)\sqrt{g(l_2) - g(l_1)} = (l_3 - l_2)\sqrt{g(l_3) - g(l_2)} \\ \vdots \\ (l_{n-1} - l_{n-2})\sqrt{g(l_{n-1}) - g(l_{n-2})} = (l_n - l_{n-1})\sqrt{g(l_n) - g(l_{n-1})} \end{cases} \quad (19)$$

Similar method of minimum walking distance as mentioned as in section 4 can be applied to find the solutions, $\{l_i, i = 1, 2, \dots, n\}$.

If people walk to the nearest bus stop, (Lesley, 1976; Wirasinghe and Ghoneim, 1981; Kikuchi, 1985) then bus stops should be located at the midpoint between boundaries, i.e. $s_i = \frac{1}{2}(l_{i-1} + l_i)$.³ Since the transit stop locations are determined we could calculate transit demand for each stop.

$$D_{bi} = D(l_i) - D(l_{i-1}) \quad (20)$$

The results of bus stop locations and travel demand are shown in Table 1. As a simplified demonstration, only the first 15 of 97 stop locations are listed. By examining Table 1, it is found that the variations of distance between two adjoining stops under inverse demand and inverse square root demand policies lie between those of equal spacing and equal demand spacing policies.

³Some papers (Vuchic and Newell, 1968; Black, 1978; Hurdle and Wirasinghe, 1980) assume people minimise travel time rather than minimise walking distance, then S_i is greater than but nearly equal to $\frac{1}{2}(l_{i-1} + l_i)$, i.e. people will have the same propensity to walk to the stop nearer their destination.

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Table 1: The First 15 Bus Stop Locations and Travel Demand Under Different Spacing Policies

I	Existing System		Equal Spacing		Equal Demand Spacing		Inverse Demand Spacing		Inverse Square Root Demand Spacing	
	l_i	D_{bi}	l_i	D_{bi}	l_i	D_{bi}	l_i	D_{bi}	l_i	D_{bi}
1	0.30	8	0.26	7	0.56	16	0.38	10	0.34	9
2	0.60	9	0.52	8	1.02	16	0.74	11	0.67	10
3	0.90	10	0.78	8	1.43	16	1.08	12	0.99	11
4	1.20	11	1.04	9	1.79	16	1.40	12	1.29	12
5	1.65	18	1.30	10	2.12	16	1.71	13	1.59	12
6	2.10	21	1.56	11	2.44	16	2.00	13	1.88	13
7	2.30	10	1.82	11	2.73	16	2.29	14	2.16	13
8	2.50	10	2.08	12	3.02	16	2.57	14	2.44	14
9	2.85	20	2.34	13	3.29	16	2.84	15	2.71	14
10	3.20	12	2.60	13	3.55	16	3.11	15	2.98	15
11	3.40	18	2.86	14	3.81	16	3.37	15	3.24	15
12	3.60	19	3.12	15	4.06	16	3.63	15	3.51	15
13	3.90	26	3.38	15	4.30	16	3.88	16	3.77	15
14	4.20	26	3.64	16	4.55	16	4.13	16	4.02	16
15	4.60	27	3.90	16	4.79	16	4.38	16	4.28	16

l_i = hinterland boundaries of stop locations (km)
 D_{bi} = numbers of boarding and alighting passengers

The theoretical model described in this paper is a planning tool only, for use in the initial stages of route planning and design. It provides first-order estimates of stop locations, based on the spacing policy adopted for a given set of circumstances. The precise locations of actual stops can only be determined after the basic route design, and then taking into account the second-order effects of such factors as intersection location, main sites of trip generation, built form of the area, geometric design of the route and traffic signal coordination. More details on the possible effects of these factors (e.g. bus stop location upstream or downstream of a signalised intersection, etc.) may be found in Institute of Traffic Engineers (1967) and Terry and Thomas (1971). Thus the results in Table 1 would have to be adjusted for the specific route in terms of these second-order factors to determine *precise* stop locations. The spacing policy model has its application in 'sketch' planning and preliminary route design.

Since transit stop locations under the different spacing policies are indicated by the model, we can calculate the transit demand for each stop, as well as total walking distance (passenger access) and bus travel time (see Ling, 1987). The results are shown in Table 2. They suggest four findings

1. The results support two theoretical derivations from sections 3 and 4: (1) the total walking distances under equal spacing and equal demand spacing are about the same; and (2) transit travel time under equal spacing policy is less than that for equal demand spacing.
2. Siting stop locations according to the minimum walking distance spacing method could reduce total walking distance in the existing system by up to 22 per cent.
3. Although the existing system has the minimum transit total travel time, the difference between this travel time and those of the theoretical spacing policies is not significant (0.9 minutes in 61 minutes, i.e. 1.5 per cent). This should be compared to the significant improvements in access time (about 22 per cent decrease in walking distance). Given Webster and Bly's (1979) study indicating that travellers value walking and waiting times as about twice as important as riding time, the model results suggest that there is considerable opportunity for service improvement.
4. The results for stop locations and transit performance under minimum walking distance are almost the same as those for the inverse demand spacing policy. These policies offer the best levels of access for passengers in this example.

Table 2: Comparison of Performance Under Different Spacing Policies

	Total Walking Distance (km)	Transit Travel Time (min)
Existing System	119.5	61.0
Minimum Walking Distance	95.7	61.9
Equal Spacing	98.0	61.6
Equal Demand Spacing	98.0	61.9
Inverse Demand Spacing	95.7	61.9
Inverse Square Root Demand	96.0	61.8

6 CONCLUSIONS

The planning and initial design of bus routes requires some assumption about the likely level of service required to meet a given level of passenger demand. The possible locations of stops are an important part of this process, and this paper has presented and examined a number of policies for stop spacing. Although a precise route design requires detailed additional information on population distribution, travel demand functions, route and land use characteristics, the following policy and planning implications may be drawn from this study:

TRANSIT STOP SPACING POLICIES

- 1 Transit ridership and operator revenue under different spacing policies are dependent on the travel demand function, which is itself related to walking distance. Transit operators would prefer equal demand spacing rather than equal distance spacing if the demand function is convex, otherwise equal distance spacing be preferred. Given a passenger demand function and bus stop spacing (or number of stops) policy, a planner could use the theoretical model to determine the stop locations that maximise operator revenue
2. Transit round trip travel time and operator cost under equal distance spacing are less than under equal demand spacing. Transit operators would prefer equal distance spacing as it offers lower operating costs. However, the differences do not appear to be significant
3. Supposing that all transit passengers access transit stops on foot, then total passenger walking distance under equal distance spacing and equal demand spacing are equal. The assumption of foot-only access is valid for many suburban bus routes.
4. If the travel demand is equally distributed along the route, then total walking distance under equal distance spacing (and, equivalently, equal demand spacing) can approach the absolute minimum total walking distance. Otherwise, the method presented in section 4 may be used to find transit stop locations that minimise total walking distance. Equal distribution of demand is an assumption that may well apply within a CBD, indicating that equal distance spacing is an appropriate policy in the CBD.

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