21st Australasian Transport Research Forum
Adelaide, September 1997

# Establishing a Fare Elasticity Regime for Urban Passenger Transport: Non-Concession Commuters

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Abstract:

The missing ingredient in many operational studies of public transport patronage prediction is a matrix of direct and cross fare elasticities which relate to specific fare classes within a choice set of fare class opportunities This paper uses a heteroskedastic extreme value choice model to relax the constant variance assumption of the multinomial logit model so that empirically realistic cross elasticities can be obtained. A combined stated preference and revealed preference data set collected in Sydney in 1995 is used to obtain a matrix of direct and cross elasticities which reflects the market environment in which commuters make choices while benefiting by a richer understanding of how travellers respond to fare profiles not always observed in the actual market, but including fare profiles which are of interest as potential alternatives to the current market offerings.

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## Introduction

Public transport operators increasingly use yield management techniques in establishing mixtures of ticket types and fare levels. In predicting the response of the market to specific fare classes and levels (eg weekly ticket), a knowledge of how various market segments respond to both the choice of ticket type within a public transport mode and the choice between modes is crucial to the outcome. In some circumstances the interest is in evaluating the patronage and revenue implications of variations in offered prices for the existing regime of fare classes; in other circumstances the interest is in changes in the fare class offerings either through deletions and/or additions of classes.

The missing ingredient in many operational studies is a matrix of appropriate direct and cross fare elasticities which relate to specific *fare classes* within a choice set of fare class opportunities. Surprisingly the research literature is relatively barren of empirical evidence that is rich enough to distinguish sensitivities to particular fare class offerings within a predefined choice set of offerings. Although there is a plethora of empirical evidence offered on direct elasticities (Oum et al 1992, Goodwin 1992), primarily treated as unweighted or weighted average fares within each public transport mode, a review of the extant literature illustrates the limited evidence on cross-elasticities. Elasticities related to specific ticket types are generally absent from the literature, and non-existent in Australia.

The cross-elasticities for rail and bus with respect to bus and rail fares are very similar, with an unweighted average value of 0.24 ± 0.06. The car-to-public transport and public transport-to-car cross elasticities however are quite different. The average cross elasticity of car demand with respect to bus fares is 0.09 ± 0.07; and with respect to train fares it is 0.08 ± 0.03. These values are significantly higher for travel to CBD destinations where the propensity to use public transport is greater (ie higher initial modal share). Authors such as Glaister and Lewis (1978) have stated that the evidence on elasticities for the impact of public transport fares on car traffic for the off-peak are largely guesswork. Twenty years on, little appears to have changed.

To obtain useful empirical elasticities applicable to particular ticket types, fare levels and mixes of ticket types offered requires site-specific empirical studies. This paper departs from the reliance on average fares, distinguishing between fare classes across two public transport modes (train, bus) and the automobile for commuting travel in the Sydney Metropolitan area. Full matrices of direct and cross share elasticities are derived for three train fare classes, three bus fare classes and car travel for commuters on non-concessionary tickets. To evaluate sizeable variations in the levels of fares in each ticket class so that operators have extended policy intelligence beyond market experience, stated choice reponses are combined with a knowledge of current modal attributes from revealed preference data to assess the ticket and mode choices made. Equivalent elasticities for non-commuters in the non-concessionary market have been obtained but are not reported herein.

The paper is organised as follows. Section 2 sets the ticket/mode choice modelling task within a microeconomic framework which guides the formulation of the indirect utility function associated with each alternative. Section 3 introduces a discrete choice model associated with the family of random utility models - heteroskedastic extreme value logit (HEVL) - which relaxes the strong assumption of constant variance in the unobserved effects to allow the cross-elasticities to break away from the equality constraint imposed in the multinomial logit model and within partitions of the popular nested logit model. Section 4 outlines the empirical context in which we source revealed and stated preference data to provide an enriched utility space for assessing behavioural responses to fare scenarios extending beyond the range observed in real markets. Section 5 presents the empirical evidence as a full matrix of direct and cross share elasticities for commuting travel. A set of conclusions highlight the major contribution of this study.

## Microeconomic specification of the indirect utility function for choice alternatives

The functional form of the conditional indirect utility expression defining the set of attributes determining the probability of selecting a mode is typically assumed to be linear additive in revealed preference models with the occasional use of logarithmic or Box-Cox transformations designed to improve the statistical 'fit' (eg Gaudry et al 1988) and occasionally specified with quadratic terms in a stated choice model with mean centered or orthogonal codes for each attribute (eg Hensher 1996). The derivation of the functional form from microeconomic theory is noticeably absent in most transportation *modal choice* applications, although examples exist in other transport applications, especially in automobile choice studies (eg Hensher et al 1992, Mannering and Winston 1985, and Train 1986).

An exception in the modal choice literature is Jara-Diaz and Videla (1989) who have derived an appropriate functional form for the indirect utility expression for a discrete mode choice model from microeconomic principles, showing that the inclusion of the income effect is accommodated by the inclusion of a quadratic term in cost and segmentation of the sample by income where the quadratic cost variable is statistically significant. It has been know for some time (but often ignored) that the inclusion of income as a separate explanatory variable serves only as a proxy for unobserved attributes of alternatives like comfort and convenience and other dimensions of taste not captured by the taste weights (Hensher 1984). Efforts to interact cost and income by dividing modal cost by the wage rate (eg Train and McFadden 1978) implicitly treats income as an endogenous variable that depends on the number of hours worked at a given wage rate in contrast to its role as an exogenous variable in an individual's budget constraint.

Without realising it, the analysts estimating stated choice models with higher order cost attributes such as a quadratic are correctly incorporating a test of the presence/absence of the income effect in the discrete choice model; unfortunately they then introduce income as an additive explanatory variable in J-1 alternatives and interpret its taste weight as a measure of the marginal utility of income; in fact the marginal utility of income is a derivative of the cost variables as shown by Jara-Diaz and Videla (1989). Inclusion of income as an income effect requires its inclusion in the indirect utility expressions for *all* alternatives.

Formally, after Jara-Diaz and Videla (1989) and Hensher (1996), for a sampled individual with a set of taste weights and income I, define a vector of non-modal trip goods X and a vector of associated prices P. The attributes of available modes, including trip cost, given by a vector $A_j$, are the observed and unobserved (by analyst) sources of utility, introduced into a utility function evaluated by an individual in arriving at a choice. Imposing the separability condition on the numeraire non-trip goods and modal alternatives defined by a set of taste-weighted modal attributes, the individual is assumed to behave as if they are maximising utility by comparing the set of modal alternatives given the separability assumption for X and each of $A_j$, j = 1,2,....., M modes:

$$\mathrm{Max}\{\max[U_1(\mathbf{X}) + U_2(\mathbf{A}_j)] | \mathbf{PX}' + c_j \le I\};\ j \in \{1,....,M\};\ \mathbf{X} \in x \qquad (1)$$

A conditional indirect utility function can be derived from (1) by the application of Roy's identity, to yield equation (2).

$$V(\mathbf{P}, I - c_j, \mathbf{A}_j) = V_1(\mathbf{P}, I - c_j) + U_2(Q_j) \qquad (2)$$

where the maximum conditional indirect utility is attributed to the chosen alternative from a mutually exclusive set of alternatives.

Jara-Diaz and Videla (1989) demonstrate that if one takes a higher order Taylor series expansion this implies solving equation (3), re-expressed as equation (4).

$$\underset{j}{\mathrm{Max}}\left[V_1(\mathbf{P},I) + \sum_{i=1}^{n-1}\frac{1}{i!}V_1^i(\mathbf{P},I)\left(-c_j\right)^i + \frac{1}{n!}V_1^n(\mathbf{P},I)\left(-c_j\right)^n + U_2\left(\mathbf{A}_j\right)\right] \qquad (3)$$

$$\underset{j}{\mathrm{Max}}\left[\sum_{i=1}^{n-1}\frac{1}{i!}V_1^i(\mathbf{P},I)\left(-c_j\right)^i + \frac{1}{n!}V_1^n(\mathbf{P},I)\left(-c_j\right)^n + U_2\left(\mathbf{A}_j\right)\right] \qquad (4)$$

From equation (4) we have identified an empirical opportunity to evaluate the dependency of mode choice on income. Adding at least a quadratic term for cost (equation (5)) will establish the potential for income dependency. In the words of Jara-Diaz and Videla (1989, 396)

> "....if a single model with utility in $c_i$, $c_i^2$, and $A_i$ were run for the whole population, a null coefficient of $c_i^2$ would be consistent with a single coefficient for $c_i$ ,......, but a significant coefficient of $c_i^2$ would be contradictory with the model, since $V_1^i$ should be a function of I. Note that I is not explicitly included in V, but significant $c_i^2$ terms for each segment would suggest the existence of a more general ..... $V(c_i, t_i, I)$ function".

$$V_i = \alpha_{0i} + \beta_{c1i}c_{1i} + \beta_{c2i}c_{2i}^2 + U_2(A_i) \tag{5}$$

Thus if $\beta_{c2i}$ is positive and statistically significant, an income effect exists and it is necessary to either segment by income so that income is affecting all alternatives in the choice set or income is accommodated in all indirect utility expressions. Having established that there is an income effect, and in the interest of maintaining a single discrete choice model, we need to introduce income into all indirect utility expressions in a way that is consistent with microeconomic theory. One appealing way is to adopt the approach promoted by Train and McFadden (1978), Hensher et al (1992), Jara-Diaz and Ortuzar (1988), Jara-Diaz and Videla (1989) and Jara-Diaz (1996) where a first order expansion of indirect utility yields a model in which money cost is divided by the expenditure rate, the latter defined as the ratio of household income to leisure (or non-work) time. This formulation represents income as purchasing power.

If one were to undertake income segmentation, then to avoid an arbitrary segmentation one could calculate the marginal utility of income and identify the variation in the marginal utility of income over the personal income space of the sampled population, yielding a number of income groupings. The marginal utility of income is given by:

$$\frac{\partial V_i}{\partial I} = \beta_{c1i} + 2\beta_{c2i}c_{2i} \tag{6}$$

## Specifying a choice model

The ticket type and mode choice model is based on the utility maximisation hypothesis which assumes that an individual's choice of ticket type conditional on mode and choice of mode is a reflection of underlying preferences for each of the available alternatives and that the individual selects the alternative with the highest utility. The utility that an individual associates with an alternative is specified as the sum of a deterministic component (that depends on observed attributes of the alternative and the individual) and a random component (that represents the effects of unobserved attributes of the individual and unobserved characteristics of the alternative).

In most mode choice models, the random components of the utilities of the different alternatives are assumed to be independent and identically distributed (IID) with a type I extreme value distribution. This results in the multinomial logit model of mode choice (McFadden, 1974). The multinomial logit model has a simple and elegant closed-form mathematical structure, making it easy to estimate and interpret. However, it is saddled with the "independence of irrelevant alternatives" (IIA) property at the individual level (Hensher and Johnson 1981, Ben-Akiva and Lerman, 1985); that is, the multinomial logit model imposes the restriction of equal cross-elasticities due to a change in an attribute affecting only the utility of an alternative i for all alternatives $j \neq i$. This property of equal proportionate change of unchanged modes is unlikely to represent actual choice behaviour in many situations. Such misrepresentation of choice behaviour can lead to misleading projections of mode share on a new or upgraded service and of diversion from existing modes.

The model developed herein falls under the category of non-IID models. Specifically, we develop a random utility model with independent, but non-identical random terms distributed with a type I extreme value distribution. This heteroskedastic extreme value model allows the utility of alternatives to differ in the amount of stochasticity (Bhat 1995). Unequal variances of the random components is likely to occur when the variance of an unobserved variable that affects choice is different for different alternatives. For example, in a mode choice model, if comfort is an unobserved variable whose values vary considerably for the train mode (based on, say, the degree of crowding on different train lines) but little for the automobile mode, then the random components for the automobile and train modes will have different variances (Horowitz, 1981). We apply this model in the current study. Once we relax the constant variance assumption we have to distinguish scale and taste, to which we now turn.

**The inseparability of taste and scale**

It has been well-known for some time that a fundamental link exists between the scale of the estimated parameters and the magnitude of the random component in all choice models based on Random Utility Theory (RUT) (see, e.g., Hensher and Johnson 1980; Ben-Akiva and Lerman 1985). Let

$$U_{iq} = V_{iq} + \varepsilon_{iq}, \tag{7}$$

where $U_{iq}$ is the unobserved, latent utility individual q associates with alternative i; $V_{iq}$ is the systematic, quantifiable proportion of utility which can be expressed in terms of observables of alternatives and consumers; and the $\varepsilon_{iq}$'s are the random or unobservable effects associated with the utility of alternative i and individual q. All RUT-based choice models are derived by making some assumptions about the distribution of the random effects; regardless of the particular assumption adopted, there is an embedded scale parameter, which is inversely related to the magnitude of the random component, that cannot be separately identified from the taste parameters.

For example, to derive the Multinomial Logit (MNL) choice model from (7), we assume that the $\varepsilon_{iq}$'s are IID Type I Extreme Value (or Gumbel) distributed. The scale parameter $\lambda \geq 0$ of the Gumbel distribution is inversely proportional to the variance of the error component, thus, $\sigma_{iq}^2 = \pi^2 / 6\lambda^2$. The fundamental identification problem of RUT-based choice models shows itself in the MNL model through the fact that the vector of parameters actually estimated from any given source of RUT-conformable preference data is actually $(\lambda\beta)$, where $\beta$ is the vector of taste parameters. This is clearly seen in the full expression of the MNL choice probability:

$$P_{iq} = \frac{\exp(\lambda V_{iq})}{\sum_{j \in C_n} \exp(\lambda V_{jq})} = \frac{\exp(\lambda \beta X_{iq})}{\sum_{j \in C_q} \exp(\lambda \beta X_{jq})}, \tag{8}$$

where $P_{iq}$ is the choice probability of alternative i for individual q, and the systematic utility $V_{iq}=\beta X_{iq}$. Since a given set of data is characterised by some value of $\lambda$, this constant is normalised to some value (say, one), and analysis proceeds as if $(\lambda\beta)$ were the taste parameters.[1]

The existing studies using data from multiple sources have all adopted a constant variance assumption within the set of alternatives associated with each data set. They have set the scale parameter to 1.0 for one data set and rescaled the other data set by a scale parameter which is constant (but possibly not equal to 1.0) across the set of alternatives. The cross elasticities remain subject to the IID assumption and hence are ill conditioned. In our study we relax the constant variance assumption and allow all scale parameters to differ within and between multiple data sets. We do this by a procedure known as a heteroskedastic extreme value random utility model. Joint estimation is essential to enable direct comparability in rescaling between the RP and SP choice models, since only one alternative across both data sets has its variance on the unobserved effects arbitrarily set to 1.0.

One way to relax the constant variance assumption requires a more complex choice model, called the heteroskedastic extreme value (HEV) model. Allenby and Ginter (1995), Bhat (1995) and Hensher (1996) have recently implemented the HEV model on a single data source. Hensher (1996a) has applied the Heteroskedastic HEV model to joint estimation of SP and RP data.

With respect to utility function (5), we assume that the data are cross-sectional (hence no temporal effects), there is no state dependence or serial dependence and tastes are homogenous. Specifically,

$$U_{iq} = \lambda_{iq}\alpha_i + \lambda_{iq}\beta X_{iq} + \varepsilon_{iq} \quad (9)$$

Now assume that the $\lambda_{iq}$ are equal to $\lambda_i$ for all individuals q; in addition, assume they are independently, but not identically, distributed across alternatives according to the Type I Extreme Value density function f(t) = exp(-t)*exp(-exp(-t)) = -F(t)*log(F(t)), where F(.) is the corresponding cumulative distribution function. If the decision rule is maximal utility, then the choice probabilities are given by

$$P_{iq} = \int_{-\infty}^{\infty} \prod_{j \neq i} F(\lambda_j)[V_{iq} - V_{jq} + \varepsilon_{iq}]\lambda_i f(\lambda_i \varepsilon_{iq}) d\varepsilon_{iq} \quad (10)$$

The probabilities are evaluated numerically as there is no closed-form solution for this single dimensional integral. The integral can be approximated, for example, using Gauss-Laguerre quadrature (Press et al 1986). (Computational experience has shown

[1]Note that the MNL model predicts random choice when $\lambda \to 0$, and approximates a step function for the alternative with maximal utility as $\lambda \to \infty$ (see Ben-Akiva and Lerman 1985) This general behaviour applies to all choice model specifications

that a 68 point approximation is sufficient to reproduce taste parameter estimates; see Greene 1996).

The heteroskedastic extreme value model nests the restrictive MNL and is flexible enough to allow differential cross-elasticities among all pairs of alternatives. It avoids the *a priori* identification of mutually exclusive market partitions of a nested MNL structure. It is parsimonious compared to the MNP model, introducing only J-1 additional parameters in the covariance matrix as opposed to the [J(J-1)/2]-1 additional parameters in the more general model (J is the total number of alternatives in the universal choice set). It also poses much less of a computational burden than the MNP, requiring only the evaluation of a one dimensional integral (independent of the number of alternatives); the MNP, of course, requires the evaluation of a J-1 dimensional integral. Importantly, in contrast to the multinomial probit model, the heteroskedastic extreme value model is easy to interpret and its behaviour is intuitive (Bhat 1995).

## The empirical context

A survey of a sample of commuters and non-commuters was undertaken in the Sydney Metropolitan Area in 1995 as part of an inquiry into the mix and level of public transport fares. Within each market segment patterns of modal and ticket use behaviour are captured to identify both current behaviour and the potential to switch to alternative modal and ticket use behaviour under a range of alternative fares policies for the government bus, ferry and train systems (Hensher and Raimond 1995).

The choice of mode and ticket type is estimated using a mixture of revealed preference (RP) and stated preference (SP) data. The RP data's strengths lie in reflecting the current state of market behaviour, whereas the SP data's strengths are that it mirrors a more robust and less restricted decision environment and presents a well-conditioned design matrix. RP data provides information on the current market equilibrium for the behaviour of interest and is useful for short term forecasting of departures from the current equilibrium. In contrast SP data is especially rich in attribute trade-off information, but is to some extent affected by the degree of 'contextual realism' that we can establish for the respondents (Hensher 1994). In deriving estimates of elasticities, the set of choice probabilities must reflect observed market behaviour (ie market shares), and hence we use the RP model enriched by the parameter estimates produced from the SP data appropriately re-scaled for each alternative when transferred to the RP model.

Sourcing revealed and stated preference data

In the survey, respondents were asked to think about the last commuter trip they made, where they went, how they travelled, how much it cost etc., then they were asked to describe another way they could have made that trip if their current mode was not available. The current behaviour provides the revealed preference data. The stated preference component of the survey varies public transport fares of their current and alternative methods of travel under a series of different pricing scenarios. Ticket prices were varied from current levels to 50% above and below current levels. Each respondent was presented with four different scenarios (see Table 1), and different respondents are presented with different combinations of scenarios. Scenarios are generated and presented such that it is possible to determine, under any fare scenario how many people will travel under each ticket and on each mode, and thus derive how sensitive people are to fare changes (elasticities). Their responses to these different scenarios are recorded in terms of what mode of transport they would use and which fare they would use.

**Table 1 Illustrative Set of Show Cards for the SP Experiment 1: Bus or Train for a Short Trip**

*You have told us that you could either use a Bus or a Train as the main form of transport to travel to the destination that we have discussed. If public transport fares changed and were priced as below, would you have used Bus or Train as the main form of transport for your trip? Which ticket type would you choose?*

| BUS FARES | | TRAIN FARES | |
|---|---|---|---|
| Single | $0.60 | Single | $0.80 |
| TravelTen<br>*(10 single trips)* | $4.00 | Off Peak Return<br>*(purchase after 9am)* | $0.90 |
| TravelPass<br>*(7 days bus/ferry)* | $8.60 | Weekly<br>*(7 days train only)* | $6.80 |
| TravelPass<br>*(7 days bus/ferry/train)* | $10.00 | TravelPass<br>*(7 days bus/ferry/train)* | $10.00 |

Given the primary emphasis is on developing a full matrix of direct and cross elasticities for mode-specific public transport fares under alternative choice sets of ticket types, we designed a sample that captured a sufficient number of travellers currently choosing each of the available modes (including car) and available ticket types in each of the market segments. Inner, middle and outer areas of Sydney are sampled in roughly equal proportions, as is each mode.

A face to face home interview was undertaken with start points generated by randomly choosing postcodes within each Statistical Local Area in Sydney. Within each postcode, a random street was chosen to be cluster sampled. The sample is "choice-based"; that is, the sampling unit is the mode (ticket type) to ensure there are enough sampled currently choosing each of the alternative modes/ticket types. This is corrected in estimation to reproduce the base market shares. In addition, all observations are weighted to the distribution of personal income for commuter demand as revealed in the 1991 Sydney Travel Survey. Although the survey included ferry and jet cat options, we have excluded them from the current analysis, since many cities have only trains and buses available as public transport competing with the automobile. Taxis were excluded from the commuter sample.

Developing the stated choice experiment

One of the difficulties associated with using a stated choice approach is the need to present individuals with an experiment which offers realistic scenarios to all respondents. Given that people use different modes and travel over greatly varying distances, it is necessary to develop a range of showcards with different modal combinations and different travel distances. Answers in the questionnaire tell the interviewer which showcards are appropriate for which respondents.

The showcards developed for this study cover every combination of main mode (car, train, bus) and have levels for short trips (less than 15 minutes), medium trips (15-30 minutes) and long trips (over 30 minutes). These times refer to the length of time spent in the main mode only, not the access, egress or waiting times. To keep the experiment and sample size to a manageable size, it was necessary to collapse the public transport ticket categories down to those most frequently used.

An experimental design was developed based on 1 car, 4 train tickets (single, off-peak, weekly and travel pass), and 4 bus tickets (single, travel ten, combined bus-ferry travel pass, and combined bus-ferry-train travel pass) — a total of 9 alternatives that are hypothetically possible for any respondent. In order to provide realistic fare scenarios to respondents, we have developed 3 different scenarios based on travel time in the main mode of travel. There is the short trip, of less that 15 minutes in the main mode, the medium length trip, of 15-30 minutes in the main mode, and the long trip, of over 30 minutes. The full range of fares in the choice experiment are summarised in Table 2.

**Empirical results**

The effective response rate was 37%, which is about average for surveys of equivalent length (Richardson et al 1995). While the full sample collected was 649 cases, not all cases had sufficient data to be suitable for modelling. As this survey exercise involved choice-based sampling, the sample is not representative of the population, but is scaled using external data to represent the population. The sample is a fairly broad representation of the Sydney population, though males and the elderly are slightly under-represented.

**Table 2 The Stated Choice Experiment Fare Categories and Levels**

| *Train: Single (Off Peak Return)* | *Low Fare* | *Current Fare* | *High Fare* |
|---|---|---|---|
| Short | $0.80 ($0.90) | $1.60 ($1.80) | $2.40 ($2.60) |
| Medium | $1.30 ($1.40) | $2.60 ($2.80) | $3.90 ($4.20) |
| Long | $1.80 ($2.00) | $3.60 ($4.00) | $5.40 ($6.00) |
| *Train: Weekly* | *Low Fare* | *Current Fare* | *High Fare* |
| Short | $6.80 | $11.50 | $18.30 |
| Medium | $9.70 | $19.40 | $29.00 |
| Long | $13.20 | $26.00 | $40.00 |
| *Train: TravelPass* | *Low Fare* | *Current Fare* | *High Fare* |
| Short | $10.00 | $20.00 | $30.00 |
| Medium | $14.00 | $28.00 | $42.00 |
| Long | $20.00 | $39.00 | $59.00 |
| *Bus: Single* | *Low Fare* | *Current Fare* | *High Fare* |
| Short | $0.60 | $1.20 | $1.80 |
| Medium | $1.30 | $2.50 | $3.80 |
| Long | $2.00 | $3.90 | $5.90 |
| *Bus: TravelTen* | *Low Fare* | *Current Fare* | *High Fare* |
| Short | $4.00 | $8.00 | $12.00 |
| Medium | $8.00 | $16.00 | $24.00 |
| Long | $16.00 | $32.00 | $48.00 |
| *Bus:TravelPass (Bus/Ferry)* | *Low Fare* | *Current Fare* | *High Fare* |
| Short | $8.60 | $17.10 | $26.00 |
| Medium | $11.70 | $23.00 | $35.00 |
| Long | $17.20 | $34.00 | $52.00 |
| *Bus: TravelPass (Bus/Ferry/Train)* | *Low Fare* | *Current Fare* | *High Fare* |
| Short | $10.00 | $20.00 | $30.00 |
| Medium | $14.00 | $28.00 | $42.00 |
| Long | $19.50 | $39.00 | $59.00 |

## Empirical models

The final models jointly estimated with 7 SP alternatives and 7 RP alternatives are presented in Table 4. Summary statistics describing the attributes of each indirect utility expression are given in Table 3, together with sample size. The mean of cost for multi-trip tickets is derived from the ticket price divided by the number of one-way trips actually undertaken by each commuter, allowing for the use of the ticket for non-commuting travel (a point often overlooked). The off-peak train single option was deleted because so few commuters choose it; in addition we had to combine the two bus travel passes (bus/ferry and bus/ferry/train) to secure enough commuters choosing one of these ticket types. McFadden (1984, page 1442) has stated that

> *"As a rule of thumb, sample sizes which yield less than thirty responses per alternative produce estimators which cannot be analysed reliably by asymptotic methods".*

**Table 3 Summary Statistics of Estimation Sample (standard deviations in parenthesis)**

| **Stated Preference Sub Sample** Alternative | Out of pocket cost ($) | Door to Door time (mins) | Captive to PT (proportion) | Car available (proportion) | Sample size |
|---|---|---|---|---|---|
| *Total Sample:* | | | | | |
| Train single | 2.89 (1.50) | 69.4 (29.6) | 0.081 | - | 540 |
| Train weekly | 2.11 (1.90) | 69.4 (29.6) | 0.081 | - | 540 |
| Train travel pass | 3.18 (1.61) | 69.4 (29.6) | 0.081 | - | 540 |
| Bus single | 2.34 (1.49) | 53.6 (26.5) | 0.119 | - | 472 |
| Bus travel ten | 1.67 (1.23) | 53.6 (26.5) | 0.119 | - | 472 |
| Bus travel pass | 1.54 (0.83) | 53.6 (26.4) | 0.119 | - | 472 |
| Car | 2.88 (2.63) | 44.9 (33.3) | - | 0.80 | 812 |
| **Revealed Preference Sub Sample** Alternative | Out of pocket cost ($) | Door to Door time (mins) | Captive to PT (proportion) | Car available (proportion) | Sample size |
| *Total Sample:* | | | | | |
| Train single | 1.64 (1.19) | 64.29 (31.1) | 0.044 | - | 272 |
| Train weekly | 2.46 (0.85) | 72.58 (28.6) | 0.317 | - | 248 |
| Train travel pass | 1.28 (1.32) | 79.60 (27.8) | 0.200 | - | 45 |
| Bus single | 2.37 (1.29) | 51.26 (24.5) | 0.074 | - | 324 |
| Bus travel ten | 1.17 (0.67) | 60.60 (32.8) | 0.160 | - | 100 |
| Bus travel pass | 1.94 (0.31) | 46.25 (20.7) | 0.333 | - | 48 |
| Car | 2.12 (2.04) | 44.88 (33.3) | - | 0.80 | 812 |

The distribution of SP costs encompass the RP cost levels although the composition of the sample in terms of captivity to public transport *given a ticket type* differs quite markedly. This is expected given that all SP fare options within a mode were offered to each respondent whereas the RP data define two alternatives - the chosen ticket (or mode) and one viable alternative. One most notable difference is in multi-use tickets (eg train weekly, travel pass and bus travel ten) where the higher incidence of RP captivity to public transport reflects reality much better than does the SP profile. Including captivity and car availability in both the SP and RP choice sets however is a valid application of contextual impacts on choices. Ceteris paribus, one expects there to be greater substitution between fare classes than between modes as a result of higher incidences of public transport captivity. Importantly this effect can be observed and modelled when ticket types are treated endogenously. Previous studies which evaluate modal choice in terms of an average fare or a single fare type per commuter are unable to represent the amount of movement between ticket types as a natural response to price changes. Such models 'force' switching between modes, overestimating the impact of fares policies on modal choice.

Fare or cost was included initially as a nonlinear effect truncated at the second-order level (equation 5), The quadratic of cost was found to be positive but not statistically significant (Table 4) under the non-constant variance assumption. Interestingly the quadratic of cost was highly significant (t-value of 9.06) in a constant variance multinomial logit model, suggesting the presence of confoundment of scale and taste weight, which is separately identified under the HEV specification. Previous studies that have investigated the presence of an income effect (eg Jara-Diaz and Ortuzar 1988, Jara-Diaz and Videla 1989) may have indeed made an incorrect interpretation of the presence or absence of an income effect because of the reliance on a simple multinomial logit model which surpresses the unobserved variance to be equal across the alternatives. Consequently we conclude the absence of an income effect in the present study; which may be intuitively sensible given the small amount of an individual's budget in Sydney devoted to commuting use-related marginal costs.

The level of service attributes represented by mode-specific door-to-door travel time are statistically significant, producing behavioural values of travel time savings at the sample mean of fare or cost ranging from \$3.36 per person hour for train and \$4.60 per person hour for car and \$4.75 per person hour for bus. These values are lower than those derived from the multinomial logit model, which produces equivalent values of respectively \$3.60, \$4.40 and \$5.40. The public transport values are lowered after allowing for differential; however the car value is increased. The MNL car value is comparable to that found in another study by Hensher for Sydney in the context of route choice, of \$4.35 per person hour (Hensher 1997). These directional results are identical to what we have found in Hensher (1966) in a commuter mode choice study for 6 capital cities. Although it is early evidence, one might be tempted to suggest that relaxing the constant variance assumption redistributes the potential time benefits of modes in favour of the automobile - the relatively inflated values of travel time savings for public transport:

> "......in the basic logit model is the result of failure to account for some unobserved influences on relative utility which are suppressed through the constant variance assumption and consequently 'distributed' to the observed effects' (Hensher 1996, 11).

If one identified an income effect, then personal income should be introduced into the utility expression for *every* alternative, in line with the theoretical requirement. To our knowledge this is the first study to combine the behavioural realism of free variance in the unobserved effects together with a theoretically defensible functional specification for the attributes in the indirect utility expressions and the richness of data fusion through mixing SP and RP choice sets. This mixture adds diversity and robustness to the process for deriving the matrix of direct and cross elasticities.

When the scale differences across all alternatives in both the SP and RP data are taken into account, the parameter estimates for each attribute common to an alternative appearing in both the SP and RP data sets should be generic. There is no microeconomic theoretical reason for treating them as data set specific which has traditionally been the assumption in both sequential and joint estimation of SP-RP models resulting in a single scale parameter attributed to all alternatives in a specific data set (e.g. Morikawa 1989, Hensher and Bradley 1993, Swait et al 1994).

**Table 4 HEV model: Joint Estimation of SP and RP Choices to evaluate the presence of an income effect**

| Attribute | Units | Alternative | SP Parameter Estimates | t-value | RP Parameter Estimates | t-value |
|---|---|---|---|---|---|---|
| One-way trip cost (or fare) | Dollars | All | -.34966 | -4.15 | -.34966 | -4.15 |
| *Trip cost squared* | Dollars | All | 0.00365 | 0.79 | 0.00365 | 0.79 |
| Door-to-door time | Minutes | Train | -.01862 | -4.44 | -.01862 | -4.44 |
| Door-to-door time | Minutes | Bus | -.02659 | -4.95 | -.02659 | -4.95 |
| Door-to-door time | Minutes | Car | -.02517 | -5.86 | -.02517 | -5.86 |
| Train single constant | | Train | 7.8198 | 3.84 | 8.7959 | 3.98 |
| Train weekly constant | | Train | 8.2091 | 3.93 | 10.319 | 4.17 |
| Train travel pass constant | | Train | 8.0665 | 3.90 | 9.2150 | 3.31 |
| Bus single constant | | Bus | 8.3482 | 4.00 | 9.4006 | 4.13 |
| Bus travel ten constant | | Bus | 8.2200 | 3.95 | 9.6701 | 4.08 |
| Bus travel pass constant | | Bus | 8.1234 | 3.94 | 9.7870 | 3.34 |
| Car constant | | Car | - | - | - | - |
| Captive to train dummy | | Train | 1.0657 | 2.42 | 1.0657 | 2.42 |
| Captive to bus dummy | | Bus | 1.4792 | 3.44 | 1.4792 | 3.44 |
| Car availability dummy | 1,0 | Car | 9.2935 | 4.09 | 9.2935 | 4.09 |
| | | | | | | |
| *Scale Parameters (StdDev in ( ))* | | | | | | |
| Train single | | Train | 0.962 (1.3336) | 3.58 | 1.515 (0.8467) | 3.73 |
| Train weekly | | Train | 0.527 (2.4358) | 2.46 | 0.340 (3.7723) | 1.33 |
| Train travel pass | | Train | 0.559 (2.2941) | 3.57 | 0.557 (2.3045) | 1.11 |
| Bus single | | Bus | 0.510 (2.5139) | 3.14 | 0.307 (4.1828) | 1.16 |
| Bus travel ten | | Bus | 0.780 (1.6448) | 3.51 | 0.353 (3.6309) | 1.18 |
| Bus travel pass | | Bus | 0.515 (2.4926) | 3.01 | 0.615 (2.0844) | 1.82 |
| Car | | Car | 3.338 (0.3842) | 4.25 | 1.283 (1.000) | Fixed |
| *Value of travel time savings *)* | | | | | | |
| Train | $/hour | 3.63 | | | | |
| Bus | $/hour | 5.05 | | | | |
| Car | $/hour | 4.79 | | | | |
| | | | | | | |
| Sample size | | 1824 | | | | |
| Log-likelihood at convergence | | -1547.64 | | | | |
| Pseudo r-squared | | .730 | | | | |

note: Value of travel time savings is calculated per one-way trip based on average number of one-way trips per ticket.

## Fare type and car cost direct and cross share elasticities

A heteroskedastic extreme value logit model relaxes the constant variance assumption of the standard multinomial logit model allowing the cross-elasticities to be alternative specific. The final set of direct and cross-elasticities are reported in Table 5. The reported results are probability weighted average estimates, derived from estimates for each individual in the sample. Each column provides one direct share elasticity and 6 cross share elasticities. A direct or cross elasticity represents the relationship between a percentage change in fare level and a percentage change in the proportion of daily one-way trips by the particular mode and ticket type.

For example, the column headed TS tells us that a 1% increase in the train single fare leads to a 0.218% reduction in the proportion of daily one-way trips by train on a single fare. In addition, this 1% single fare increase leads to a 0.001% higher proportion of one-way trips on a train travel pass and 0.001% increase in one-way trips on a train weekly ticket.

The set of fare elasticities are based on the use of the SP parameter estimates for fare and cost, rescaled into the RP model which provides the choice probabilities and fare (or car cost) attribute levels. Since the HEV model does not have a closed form solution, the elasticity formula is complex requiring the derivation of integrals by quadrature for equation 10. For completeness and comparison we have reported the direct and cross elasticities from the SP model and the MNL direct elasticities (noting that the cross elasticities for an MNL model are uninformative).

### Table 5 Direct and Cross Share Elasticities

Note: Elasticities relate to the total ticket price, not price per one-way trip. SP direct and cross elasticities from the HEV model are in parenthesis. The MNL direct elasticities are in square brackets from the RP and SP choice sets respectively; The interpretation for a specific fare class is obtained under each column heading

| | TS | TW | TP | BS | BT | BP | Car |
|---|---|---|---|---|---|---|---|
| Train single (TS) | **-.218** (-.702) [-.161-.517] | .001 (.289) | .001 (.149) | .057 (.012) | .005 (.015) | .005 (.009) | .196 (.194) |
| Train weekly (TW) | .001 (.213) | **-.093** (-.635) [-.057,-.313] | .001 (.358) | .001 (.025) | .001 (.024) | .006 (.019) | .092 (.229) |
| Train travel pass (TP) | .001 (.210) | .001 (.653) | **-.196** (-1.23) [-.111,-.597] | .001 (.023) | .012 (.022) | .001 (.017) | .335 (.218) |
| Bus single (BS) | .067 (.023) | .001 (.053) | .001 (.031) | **-.357** (-.914) [-.217,-.418] | .001 (.248) | .001 (.286) | .116 (.096) |
| Bus travel ten (BT) | .020 (.020) | .004 (.037) | .002 (.023) | .001 (.206) | **-.160** (-.462) [-.083,-.268] | .001 (.163) | .121 (.090) |
| Bus travel pass (BP) | .007 (.025) | .036 (.063) | .001 (.034) | .001 (.395) | .001 (.290) | **-.098** (-.700) [-.072,-.293] | .020 (.103) |
| Car (C1) | .053 (.014) | .042 (.023) | .003 (.013) | .066 (.009) | .016 (.011) | .003 (.006) | **-.197** (-.138) [-.130,-.200] |

The results offer many implications. The differences in direct elasticities between the SP and RP choice sets reflects the different probabilities of choice. As is well known, although often ignored, studies which derive elasticities from stand-alone SP models tend to get exaggerated switching propensities, which arises from the accumulating evidence that respondents have a tendency to exaggerate their stated responses, no matter how well the choice experiment is designed. Since an elasticity calculation uses three inputs - a predicted choice probability, a taste weight (and a scale parameter in an HEV model) and an attribute level, the appropriate probabilities must come from the RP model. The RP direct elasticities for public transport are lower than the SP equivalences; however since the results are driven primarily by probability differences, some elasticities must be higher for the SP model. This is the case for the car mode; explained by the fact that the SP percentage choosing the car is less than the actual market share.

For direct elasticities, sensitivity within the commuter rail and bus markets decreases as we move from a single ticket through to multiple-trip tickets. This has interesting implications for a fares policy - increasing the price of a multi-use ticket offers higher revenue growth prospects for small losses of patronage than is the case for single tickets. The cross elasticities suggest that there is more movement between modes for a given fare class than between fare classes within modes. The strongest cross-mode substitution occurs between train and bus single tickets, although it is not symmetrical, with cross elasticities of 0.067 and 0.057 for train to bus and bus to train respectively. The largest cross elasticity is 0.335 for the switch from car to train travel pass in the event of a price increase in car use. The extant empirical evidence suggests that trains have more success in attracting commuters out of cars than do buses. A travel pass per trip is the best value for money train fare (see Table 3) where the price per one-way trip is $1.28 compared to $1.64 for a train single and $2.46 for a travel ten ticket. All the cross elasticities associated with car operating costs are sizeable compared to the other modal switching contexts. Interestingly, changes in public transport fares across all ticket categories has less of an impact on car use than a change in car costs has on public transport use.

A comparison of the HEV and MNL revealed preference elasticities shows a systematically lower set of direct elasticity estimates for all alternatives in the MNL model; thus on the one hand we might conclude that an SP model tends to produce lower elasticities than its RP counterpart where the SP choice probabilities are higher than the RP probabilities; and MNL direct elasticity estimates tend to be lower than their HEV counterparts in both RP and SP models. The implications, if generalisable, is that all previous studies which have used an MNL framework and/or a stand-alone SP model specification have made sizeable errors in their estimation of direct share elasticities. Since the majority of travel choice studies have adopted this framework, the findings are quite troublesome for the extant literature.

## Conclusions

The results reported here are based on estimation of stated and revealed choice data where the variances of the unobserved components of the indirect utility expressions associated with each of the 7 ticket/mode alternatives are different. The taste weights attached to fares in the stated choice model have been rescaled by the ratio of the variances associated with fare for a particular alternative across the two model systems so that the richness of the fare data in the stated choice experiment enriches the market model. The resulting matrix of direct and cross elasticities reflects the market environment in which commuters make choices while benefiting by an enhanced understanding of how travellers respond to fare profiles not always observed in the actual market, but including fare profiles which are of interest as potential alternatives to the current market offerings.

A better understanding of market sensitivity to classes of tickets is promoted as part of the improvement in management practices designed to improve fare yields. In this paper we have examined a number of approaches to estimating a matrix of direct and cross price share elasticities, and provide for the first time a published complete asymmetric matrix. The Institute of Transport Studies has developed a decision support system (titled 'Fares Fair') in which the matrix of elasticities are the behavioural base. Public transport operators in NSW are using the DSS to evaluate the implications on revenue and patronage of alternative fare scenarios in respect of mixture of ticket types and levels of fares. Extensions of the current paper are in progress which accommodate new ticket types as well as adjust the share elasticities to provide approximate demand elasticities for both commuter and non-commuter travel.

## Acknowledgments

This paper draws on a survey undertaken on behalf of the Government Pricing Tribunal of NSW. Permission to use parts of this study is appreciated. I am indebted to Jim Cox, Michael Seery and Tom Parry for their support. Tim Raimond and Jenny King of ITS contributed substantially to the original study. Discussions with Bill Greene, Sergio Jara-Diaz, Chandra Bhat, Jordan Louviere, John Dodgson and Joffre Swait are appreciated.

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