

## Applications of Fuzzy Systems in the Environmental Evaluation of Transport Projects

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### Abstract:

This paper outlines some of the significant features of fuzzy logic as a framework for the development of methods for the evaluation of alternative transport projects, specifically the identification of preferred transport plans, routes or modal systems. Fuzzy logic-based methods have potential to incorporate the uncertainty and imprecision characteristic of transport projects. Uncertainty is characterised in deterministic rather than in statistical terms. In particular, a method involving multiple fuzzy rules (conditional propositions, implications), where antecedents relate to environmental factors or impacts and the consequent is a measure of preference associated with those factors, is illustrated in the context of the evaluation of road projects. Each factor or impact is defined as a fuzzy subset of the set of projects. The method facilitates the use of linguistic descriptions of project performance (such as 'high', 'medium', 'low', 'very low', 'very high', etc) with respect to environmental factors. Such soft factors may be combined with conventional quantitative data. The need to aggregate linguistically expressed judgements, or to combine soft and hard data is a common feature of environmental project evaluation. A simple example is given involving four road projects and six impacts (travel-time saved, social impact, flora/fauna impact, noise impact, air quality impact, capital cost).

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## Introduction

Major transport investment projects frequently engender widespread community concern and opposition. Though transport projects provide significant benefits to some sections of the community, other sections of the community and the wider environment often suffer significant disbenefits. Rarely are the disbenefits of transport projects fully accounted for since it is common for project evaluation to be based predominantly on economic criteria, in particular, travel-time savings, construction cost, and operating cost (Queensland Transport, 1993). However, it is increasingly recognised that this traditional approach towards evaluation characteristic of engineers and economists is deficient, and that broader, more encompassing evaluation is required taking account of non-efficiency criteria and non-monetary impacts. Examples of non-efficiency criteria and non-monetary impacts or environmental factors have been given in Lane (1978), OECD (1994), and NCHRP (1994). Formal assessment of the diverse range of impacts associated with transport projects has been limited. One notable exception is a recent analysis of transport infrastructure proposals by Mitchell McCotter Consultants in relation to 16 transit options associated with the South West Area Transit (SWAT) Study in Perth (Snashall, 1994).

Substantial development of formal evaluation methodology in recent years has resulted in an overwhelming variety of approaches. It is believed that formal methodology should form the basis of a broader evaluation of transport projects undertaken by transport authorities both for one-off projects and in the context of long term strategic planning. Some recent examples of formal approaches to the evaluation of transport projects include Pearman *et al* (1989), Gomes (1989), Won (1990), Pearman *et al* (1990), and Teng and Tzeng (1994).

Recently *fuzzy evaluation methods* have been proposed which more adequately acknowledge the uncertainty and imprecision characteristic of transport project evaluation. In this context, imprecision is of a non-random (deterministic) or ambiguous nature rather than of a random or statistical nature (Kosko, 1992). In project evaluation, a useful expression of such deterministic uncertainty is in terms of a value (e.g. 'low') of a linguistic variable (e.g. 'cost'). 'Low cost' may be regarded as the label of a fuzzy set. A *fuzzy set* (or, more precisely, *fuzzy subset* (Kaufmann, 1975)) is a set whose elements belong to the set in varying degrees. More formally a fuzzy subset  $A$  in a set  $X$  (a collection of objects denoted generically by  $x$ ) is a set of ordered pairs  $A = \{(x, A(x)) | x \in X\}$  where  $A(x)$  is called the membership value or grade of membership (Zadeh, 1965; Terano *et al*, 1987). When  $X$  is discrete,  $A$  is commonly written as  $\sum A(x) | x$  where the summation sign denotes the collection of all points  $x \in X$  with associated membership value  $A(x)$ .

Some fuzzy approaches to the evaluation of projects have been examined elsewhere, for example, Smith (1994). The development of fuzzy evaluation methods is consistent with increasing interest in the application of fuzzy methods to transport planning in general. See, for example, the special issue of *Transportation Planning and Technology* (2/17, 1993).

This paper presents a method of transport project evaluation based on fuzzy multiple input, single output (MISO) systems involving multiple 'if... then' rules (conditional propositions, implications) where antecedents are impacts/factors of environmental significance and the consequent is a measure of preference associated with those impacts/factors. Factors may be either *quantitative* (measured in crisp, precise, numerical terms) or *qualitative* (subjectively estimated involving, for example, expert opinion, rule of thumb, etc (Juang *et al.*, 1993).

### Fuzzy logic

Fuzzy logic is based on the theory of fuzzy subsets incorporating fuzzy subsets into the framework of multivalued logic. Fuzzy logic provides foundations for approximate reasoning with imprecise propositions analogous to the foundation provided by two-valued quantified predicate logic for precise reasoning. Fuzzy logic allows the use of fuzzy predicates (e.g. 'old', 'young'), fuzzy quantifiers (e.g. 'many', 'few'), fuzzy truth values (e.g. 'very true', 'rather true'), and fuzzy hedges or modifiers (e.g. 'very', 'fairly') (Zadeh, 1975). In fuzzy logic an implication or 'if... then' rule may be expressed as 'if **A** then **B**' or, more formally, 'if **V** is **A** then **U** is **B**', where **V** and **U** are linguistic variables and **A** and **B** are fuzzy subsets of base sets, **X** and **Y**, respectively. An important fuzzy implication inference rule is, given **V** is **A**<sub>0</sub>, where **A**<sub>0</sub> is a fuzzy subset of **X**, then infer **U** = **B**<sub>0</sub> where **B**<sub>0</sub> is a fuzzy subset of **Y**. This inference is referred to as *generalised modus ponens* which reduces to *modus ponens* when **A**<sub>0</sub> = **A** and **B**<sub>0</sub> = **B**. The inferred fuzzy subset, **B**<sub>0</sub>, can be found by the composition **B**<sub>0</sub> = **A**<sub>0</sub> ◦ **R** where **R** is a fuzzy relation, for example, **R** = (**A** × **B**) ∪ (¬**A** × **Y**) with  $R(x,y) = (A(x) \wedge B(y)) \vee (1 - A(x))$  representing a fuzzy implication, **A**-**B** (Zadeh, 1973; Ross, 1993). The implication **R** = **A**-**B** = (**A** × **B**) ∪ (¬**A** × **Y**) reduces to the implication operator of traditional propositional logic when  $A(x), B(y) \in \{0,1\}$ ; that is **R** = **A**-**B** = 0 iff  $A(x) = 1$  and  $B(y) = 0$ , otherwise **R** = **A**-**B** = 1. The fuzzy composition, '◦', is often taken as a max-min composition defined as  $B_0(y) = \bigvee_{x \in X} (A_0(x) \wedge R(x,y))$ .

Alternative implication operators are the *correlation minimum*, **R** = **A**×**B**,  $R(x,y) = A(x) \wedge B(y)$  (Kosko, 1992; Mamdani, 1976) and the *correlation product*, **R** = **A**×**B**,  $R(x,y) = A(x)B(y)$  (Kosko, 1992; Larsen, 1980). The correlation minimum inference scheme is  $B_0(y) = \bigvee_x (A_0(x) \wedge R(x,y)) = \bigvee_x (A_0(x) \wedge A(x) \wedge B(y)) = \tau \wedge B(y)$ , where  $\tau = \bigvee_x (A_0(x) \wedge A(x))$  is the degree to which **A**<sub>0</sub> and **A** overlap.  $\tau$  may be interpreted as a measure of the degree to which the rule 'if **A** then **B**' is fired for a specific **A**<sub>0</sub>. Note that when **A**<sub>0</sub> = **A**, then **B**<sub>0</sub> = **B**, if **A** is normal ( $\exists$  at least one element  $x \in X$  such that  $A(x) = 1$ ). Often **A**<sub>0</sub> is assumed to be a *fuzzy singleton*, that is,  $A_0(x) = 1$  if  $x = x_0$ ,  $A_0(x) = 0$ , if  $x \neq x_0$ ,  $x \in X$ . When **A**<sub>0</sub> is a fuzzy singleton,  $\tau = \bigvee_x (A_0(x_0) \wedge A(x_0)) = 1 \wedge A(x_0) = A(x_0)$ . The correlation minimum truncates fuzzy subset **B** at  $\tau$ , the truth of the premise.

The correlation-product inference scheme is  $B_0(y) = \tau B(y)$  where  $\tau = \bigvee_x (A_0(x) \wedge A(x))$

Again, when  $A_0 = A$ ,  $B_0 = B$  ( $A$  normal). The correlation product inference scheme preserves the shape of fuzzy subset  $B$  but scales  $B$  to the truth of the premise,  $\tau$ . Note that neither the correlation minimum, nor the correlation product, satisfy the properties of the implication in traditional propositional logic, as does, for example,  $R(x,y) = (A(x) \wedge B(y)) \vee (1 - A(x))$

### Fuzzy Systems

In general,  $m$  'if...then' rules, each with  $n$  antecedents, may be expressed as follows

If  $V_1$  is  $A_{11}$  and  $V_2$  is  $A_{12}$  and ... and  $V_n$  is  $A_{1n}$  then  $U$  is  $B_1$  else  
 If  $V_1$  is  $A_{21}$  and  $V_2$  is  $A_{22}$  and ... and  $V_n$  is  $A_{2n}$  then  $U$  is  $B_2$  else

...  
 If  $V_1$  is  $A_{m1}$  and  $V_2$  is  $A_{m2}$  and ... and  $V_n$  is  $A_{mn}$  then  $U$  is  $B_m$

where  $V_1, V_2, \dots, V_n$  and  $U$  are linguistic variables,  $A_{i1}$  is a fuzzy subset of  $X_1$  (i.e. linguistic value of  $V_1$ ),  $A_{i2}$  is a fuzzy subset of  $X_2$  (i.e. linguistic value of  $V_2$ ), etc,  $B_i$  is a fuzzy subset of  $Y$  (i.e. linguistic value of  $U$ ), and 'else' is interpreted variously, as for example, 'and', or 'or' (Yager, 1981; Lee, 1990). Assume that the dimensionality (cardinality) of base sets is  $\dim(X_j) = p_j$  ( $j = 1, \dots, n$ ) and  $\dim(Y) = q$ . Given  $V_1$  is  $A_{01}$  and  $V_2$  is  $A_{02}$  and ... and  $V_n$  is  $A_{0n}$ , infer  $U$  is  $B_0$ . Such systems of 'if... then' rules each with multiple antecedents and a single consequent are referred to in the fuzzy logic control context as *multiple input, single output* (MISO) systems (Lee, 1990) or as a *fuzzy system* (Kosko, 1992). Inputs to the system are often assumed to be *fuzzy singletons* such that  $A_{0j}(x_j) = 1$  if  $x_j = x_{0j}$ ,  $A_{0j}(x_j) = 0$ , if  $x_j \neq x_{0j}$ ,  $x_j \in X_j$ . Thus,  $\tau_i = A_{i1}(x_{01}) \wedge A_{i2}(x_{02}) \wedge \dots \wedge A_{in}(x_{0n})$  though for fuzzy inputs  $A_{0j}$  ( $j = 1, \dots, n$ ),  $\tau_i = \bigvee_{x_{11}} (A_{01}(x_{11}) \wedge A_{i1}(x_{11})) \wedge \bigvee_{x_{22}} (A_{02}(x_{22}) \wedge A_{i2}(x_{22})) \wedge \dots \wedge \bigvee_{x_{nn}} (A_{0n}(x_{nn}) \wedge A_{in}(x_{nn}))$ .

Fuzzy systems are in a sense parallel processors (Cox, 1992; 1995). Given input values, all rules that have any truth in their premises will fire and contribute to the output fuzzy set. Fuzzy systems have been predominantly used in the context of control (de Silva, 1995; Driankov *et al*, 1993), though other applications in the context of evaluation have appeared (Levy *et al*, 1991; Levy and Yoon, 1995)

The min/max fuzzy system (Mamdani method) has output set  $B_{0i}$  inferred by  $i$ th rule as  $B_{0i}(y) = \tau_i \wedge B_i(y)$  where  $\tau_i = \bigwedge_{j=1..n} A_{ij}(x_{0j})$  and where the inputs  $A_{0j}$  are crisp values  $x_{0j}$  ( $j = 1, \dots, n$ ). Individual rules are then aggregated by  $B_0(y) = \bigvee_{i=1..m} B_{0i}(y) = \bigvee_{i=1..m} (\tau_i \wedge B_i(y))$ . Defuzzification of  $B_0 = \{B_0(y)|y\}$  produces a crisp output for the fuzzy system. Many defuzzification methods exist

The COA (Centre of Area, Centre of Gravity) defuzzification method is as follows

$$\text{DEFUZZ}(\mathbf{B}_0) \equiv \text{COA}(\mathbf{B}_0) = \frac{\sum_{r=1}^q B_0(y_r)y_r}{\sum_{r=1}^q B_0(y_r)}$$

where  $\mathbf{B}_0 = \{B_0(y_1)/y_1, B_0(y_2)/y_2, \dots, B_0(y_q)/y_q\}$  Other defuzzification methods include the MOM (Mean of Maxima) method and the BADD (BASic Defuzzification Distribution) method (Filev and Yager, 1991) The BADD defuzzification method yields the COA method and MOM method as special cases

When the centre of area/gravity (COA) method is used for defuzzification, Mizumoto (1991) refers to the min/max fuzzy system (Mamdani method) as the min/max gravity method. However, numerous alternative structures are possible For example, Mizumoto (1991) proposes a product/sum gravity method as more appropriate. The product/sum gravity method has output set  $\mathbf{B}_0$ , inferred by  $i$ th rule as  $B_{0i}(y) = \tau_i B_i(y)$ , where  $\tau_i = \prod_{j=1}^n A_{ij}(x_{0j})$  and where the inputs  $A_{0j}$  are crisp values  $x_{0j}$ , ( $j = 1, \dots, n$ ). Individual rules are then aggregated by  $B_0(y) = \sum_{i=1}^m B_{0i}(y) = \sum_{i=1}^m (\tau_i B_i(y))$  (or by an average,  $B(y) = (1/m) \sum_{i=1}^m B_{0i}(y) = (1/m) \sum_{i=1}^m (\tau_i B_i(y))$ ). It is also possible to let  $\tau_i = \wedge_{j=1}^n A_{ij}(x_{0j})$  (Mizumoto, 1994). Defuzzification of  $\mathbf{B}_0$  produces an identical crisp output for the fuzzy system regardless of whether the sum or average is taken Mizumoto (1994) proposes other methods, for example, the product/algebraic sum gravity method and the bounded product/bounded sum gravity method

More flexible structures have been presented for fuzzy systems models involving ordered weighted averages (OWAs) (Yager and Filev, 1994a). Soft OWAs, S-OWA-OR (OR-like) and S-OWA-AND (AND-like) aggregation operators have been proposed (Yager and Filev, 1994b). Soft OWAs may be used to generalise the min/max (Mamdani) method which involves the correlation minimum for implementing the logical 'and' between inputs and outputs. If the correlation product operator is used, then it can be shown that a special case of the generalisation is

$$B_0(y) = (1/m) \sum_{i=1}^m (\tau_i B_i(y))$$

where  $\tau_i = \wedge_{j=1}^n A_{ij}(x_{0j})$ . Using the COA defuzzification,

$$\text{COA}(\mathbf{B}_0) = \frac{\sum_{i=1}^m \tau_i [B_i][Y]^T}{\sum_{i=1}^m \tau_i [B_i][I]^T}$$

where  $[B_i]$  is the consequent fuzzy subset of membership values for rule  $i$  represented as a row vector,  $[Y]$  is a row vector of the elements of the consequent base set  $Y$ , and  $[I]$  is a row vector of dimension  $q$  with elements unity. The COA of each consequent fuzzy subset,  $\mathbf{B}_i$  ( $i = 1, \dots, m$ ) is

$$\text{COA}(\mathbf{B}_i) = [B_i][Y]^T/[B_i][I]^T$$

and  $S_i = [B_i][I]^T$  is the power of fuzzy subset  $B_i$ . Thus  $COA(B_i)S_i = [B_i][Y]^T$  and

$$COA(B_0) = \frac{\sum_{i=1}^m \tau_i COA(B_i)S_i}{\sum_{i=1}^m \tau_i S_i}$$

The crisp output,  $COA(B_0)$ , thus depends only on the firing strengths,  $\tau_i$ , since  $COA(B_i)$  and  $S_i$  are characteristics of the output fuzzy subset  $B_i$ . This method has been called *direct fuzzy reasoning* (Yager and Filev, 1994b). The above expression is identical to that of the *additive fuzzy system* (Kosko, 1992).

### Fuzzy systems in transport project evaluation

Commonly the base sets  $X_j$  ( $j = 1, \dots, n$ ) and  $Y$  in fuzzy systems involve natural scales. However, in the context of the environmental project evaluation, natural scales for the base sets of antecedents  $X_1, X_2, \dots, X_n$  are not always available, or if available, then data is often not forthcoming. For this reason, it is possible to let  $X_1, X_2, \dots, X_n = P = \{P_1, P_2, \dots, P_T\}$ , the set of projects.  $T$  is the number of projects. Thus projects are assessed relative to each other (with respect to each impact/factor), rather than in absolute terms. Each antecedent in rule  $i$  represents the desired performance of projects with respect to an environmental impact/factor. The consequent is a measure of preference for the performance of projects with respect to the antecedents of rule  $i$ .

Preference may be defined as a fuzzy subset of a discrete base set  $Y = \{y_1, y_2, \dots, y_{11}\} = \{0.0, 0.1, \dots, 1.0\}$ , so that **high** preference, for example, might be represented as a fuzzy subset of  $Y$

$$\text{high} = \{ \text{high}(y_1)|y_1, \text{high}(y_2)|y_2, \dots, \text{high}(y_{11})|y_{11} \}$$

Given that the base set of each antecedent in each rule  $i$  ( $i=1, \dots, m$ )

If  $V_1$  is  $A_{i1}$  and  $V_2$  is  $A_{i2}$  and ... and  $V_n$  is  $A_{in}$  then  $U$  is  $B_i$

is  $P$ , then the conjunction of antecedents ' $V_1$  is  $A_{i1}$  and  $V_2$  is  $A_{i2}$  and ... and  $V_n$  is  $A_{in}$ ' ( $= A_i$ , say) may be expressed as the algebraic product

$$\tau_i(p) = \prod_{j=1}^n [A_{ij}(p)], p \in P$$

where the firing strength,  $\tau_i(p)$ , is explicitly indexed by project  $p \in P$ . The consequent in rule  $i$  is then given by the product  $B_{0i}(p,y) = \tau_i(p)B_i(y)$ ,  $p \in P, y \in Y$ .

Weights may be associated with antecedents in order to reflect the fact some antecedents are more important than others. As such, the 'if...then' rules might be represented as

If  $V_1$  is  $A_{i1}^{w_{i1}}$  and  $V_2$  is  $A_{i2}^{w_{i2}}$  and . . . and  $V_n$  is  $A_{in}^{w_{in}}$  then  $U$  is  $B_i$

where  $w_{ij}$  is the weight assigned to antecedent  $j$  in rule  $i$ . A rule with weighted antecedents is termed an 'elastic' fuzzy rule (Werbos, 1993). Thus

$$\tau_i(p) = \prod_{j=1}^n [A_{ij}(p)]^{w_{ij}}, p \in P$$

Since in practice, the number of antecedents may vary from rule to rule, a modification to the calculation of  $\tau_i(p)$  counteracts the tendency for  $\tau_i(p)$  to decrease in value as the number of antecedents increases (Tseng and Teo, 1994)

$$\tau_i(p) = \left[ \prod_{j \in n_i} [A_{ij}(p)]^{w_{ij}} \right]^{1/\sum_j w_{ij}}, p \in P$$

Thus the product and summation are taken over only those antecedents included in rule  $i$ ,  $j \in n_i$  where  $n_i$  is the set of antecedents in rule  $i$ . The consequent for rule  $i$  is then the product  $B_{0i}(p,y) = \tau_i(p)B_i(y)$ ,  $p \in P$ ,  $y \in Y$ , as above.

Rules may also involve the disjunction of weighted antecedents as follows

If  $V_1$  is  $A_{i1}^{w_{i1}}$  or  $V_2$  is  $A_{i2}^{w_{i2}}$  or . . . or  $V_n$  is  $A_{in}^{w_{in}}$  then  $U$  is  $B_i$

In this case, a generalisation of the algebraic sum may be used (Zole and Zimmermann, 1979; Keller and Krishnapuram, 1994)

$$\tau_i(p) = 1 - \prod_{j=1}^n [1 - A_{ij}(p)]^{w_{ij}}, p \in P$$

Again, since the number of antecedents may vary from rule to rule, a modification below counteracts the tendency for  $\tau_i(p)$  to increase in value as the number of antecedents increases

$$\tau_i(p) = 1 - \left[ \prod_{j \in n_i} [1 - A_{ij}(p)]^{w_{ij}} \right]^{1/\sum_j w_{ij}}, p \in P$$

The consequent for rule  $i$  is again the product  $B_{0i}(p,y) = \tau_i(p)B_i(y)$ ,  $p \in P$ ,  $y \in Y$ .

The 'else' in the rule-based system is interpreted as 'or' so that  $B_0 = \cup_{i=1}^m B_{0i}$  and  $B_{0i}(p,y) = \cup_{i=1}^m \{B_{0i}(p,y)\} = \cup_{i=1}^m \{\tau_i(p)B_i(y)\}$ . Each row of  $B_0$  is defuzzified using some appropriate defuzzification method. Alternatively, aggregation may be additive

$$B_0(p,y) = (1/m) \sum_{i=1}^m (\tau_i(p)B_i(y)).$$

Using the COA defuzzification for given  $p \in P$

$$COA(\mathbf{B}_0(p)) = \frac{\sum_{i=1}^m \tau_i(p) [\mathbf{B}_i][\mathbf{Y}]^T}{\sum_{i=1}^m \tau_i(p) [\mathbf{B}_i][\mathbf{I}]^T}$$

where  $[\mathbf{B}_i]$ ,  $[\mathbf{Y}]$ , and  $[\mathbf{I}]$  are as above.

The COA of each consequent fuzzy subset,  $\mathbf{B}_i$  ( $i = 1, \dots, m$ ) is as previously given,  $COA(\mathbf{B}_i) = [\mathbf{B}_i][\mathbf{Y}]^T/[\mathbf{B}_i][\mathbf{I}]^T$ , so that

$$COA(\mathbf{B}_0(p)) = \frac{\sum_{i=1}^m \tau_i(p) COA(\mathbf{B}_i) S_i}{\sum_{i=1}^m \tau_i(p) S_i}$$

Again, the crisp output,  $COA(\mathbf{B}_0(p))$ , depends only on the firing strengths,  $\tau_i(p)$ , since  $COA(\mathbf{B}_i)$  and  $S_i$  are characteristics of  $\mathbf{B}_i$ .

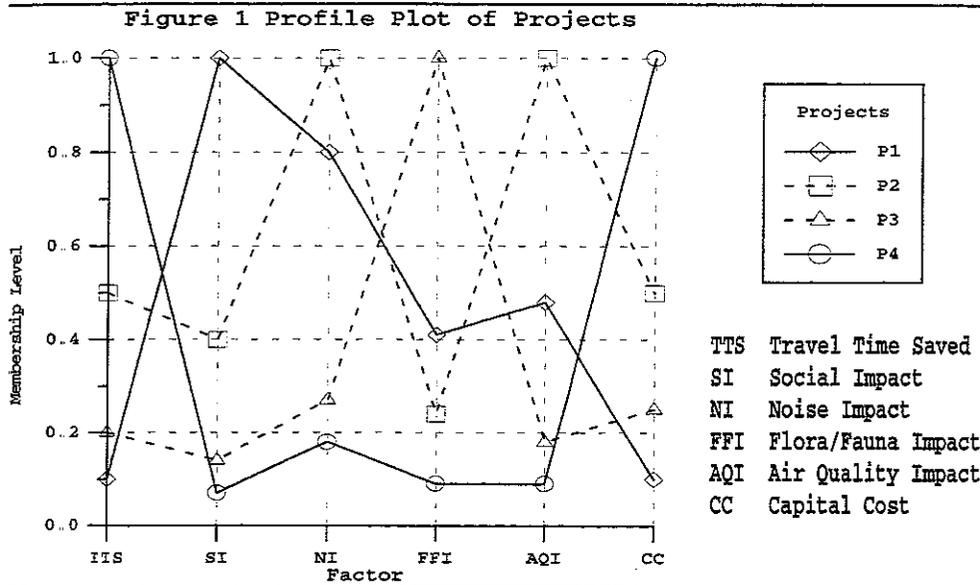
### Example

As an example, consider four transportation projects (alternative route alignments) assessed against six impacts/factors  $F_1$  (travel-time savings),  $F_2$  (social impact),  $F_3$  (noise impact),  $F_4$  (flora/fauna impact),  $F_5$  (air quality impact), and,  $F_6$  (capital cost). Let the impacts/factors be measured on the base set  $P = \{P_1, P_2, P_3, P_4\}$  of projects as follows

HTTS	=	{0.10 $P_1$ , 0.50 $P_2$ , 0.20 $P_3$ , 1.00 $P_4$ }
LSI	=	{1.00 $P_1$ , 0.40 $P_2$ , 0.14 $P_3$ , 0.07 $P_4$ }
LNI	=	{0.80 $P_1$ , 1.00 $P_2$ , 0.27 $P_3$ , 0.18 $P_4$ }
LFFI	=	{0.41 $P_1$ , 0.24 $P_2$ , 1.00 $P_3$ , 0.09 $P_4$ }
LAQI	=	{0.48 $P_1$ , 1.00 $P_2$ , 0.18 $P_3$ , 0.09 $P_4$ }
LCC	=	{0.10 $P_1$ , 0.50 $P_2$ , 0.25 $P_3$ , 1.00 $P_4$ }

where HTTS = High Travel-Time Savings, LSI = Low Social Impact, LNI = Low Noise Impact, LFFI = Low Flora/Fauna Impact, LAQI = Low Air Quality Impact and LCC = Low Capital Cost. Here, it is assumed that  $F_1$  (travel-time savings measured in hours) and  $F_6$  (capital cost measured in dollars) are *quantitative* factors with values ( $P_1 = 1000$ ,  $P_2 = 5000$ ,  $P_3 = 2000$ ,  $P_4 = 10000$ ) and ( $P_1 = 10000$ ,  $P_2 = 2000$ ,  $P_3 = 4000$ ,  $P_4 = 1000$ ), respectively. For quantitative impacts/factors, membership values are developed as  $\phi_j/\phi_j^{\max}$  for positive impacts (higher values more desired) and  $\phi_j^{\min}/\phi_j$  for negative impacts (lower values more desired), where  $\phi_j$  is the outcome of project  $P_i$  with respect impact  $F_j$ ,  $\phi_j^{\max} = \bigvee_{i=1}^4 \{\phi_{ij}\}$ , and  $\phi_j^{\min} = \bigwedge_{i=1}^4 \{\phi_{ij}\}$  ( $j = 1, \dots, 6$ ). Thus, for positive factor, **HTTS = High Travel-Time Savings** = {0.1 $P_1$ , 0.5 $P_2$ , 0.2 $P_3$ , 1.0 $P_4$ } and, for negative factor, **LCC = Low Capital Cost** = {0.1 $P_1$ , 0.5 $P_2$ , 0.25 $P_3$ , 1.0 $P_4$ }. Membership values for *qualitative* impacts/factors ( $F_2 - F_5$ ) were developed using reciprocal pairwise comparisons (Saaty, 1980). **Figure 1** shows a polygonal profile plot of the projects. Thus  $P_4$  is a minimally environmentally sensitive, project emphasising predominantly engineering/economic factors. The other projects  $P_1 - P_3$  satisfy the environmental factors to varying extents but

perform less satisfactorily with respect to engineering/economic factors.



Assume a rule-based fuzzy system consisting of  $m = 4$  rules as follows

- If  $V_1$  is **very\_high** and  $V_2$  is **low** and  $V_3$  is **low** and  $V_4$  is **low** and  $V_5$  is **low** and  $V_6$  is **fairly\_low** then  $U$  is **very\_high**
- If  $V_1$  is **very\_high** and  $V_2$  is **very\_low** and  $V_3$  is **very\_low** and  $V_4$  is **very\_low** and  $V_5$  is **very\_low** and  $V_6$  is **very\_low** then  $U$  is **very\_very\_high**
- If  $V_1$  is **high** and  $V_2$  is **low** and  $V_3$  is **low** and  $V_4$  is **low** and  $V_5$  is **fairly\_low** and  $V_6$  is **fairly\_low** then  $U$  is **moderate**
- If  $V_2$  is **not\_fairly\_low** or  $V_3$  is **not\_fairly\_low** then  $U$  is **fairly\_low**

Note that rules  $i = 1, 2, 3$  involve conjunctions of antecedents and rule  $i = 4$  involves a disjunction of antecedents.  $V_j$  is a linguistic variable associated with impact/factor  $F_j$  ( $j = 1, \dots, 6$ ) and **very\_high** is defined as  $\text{high}^2$ , **very\_low** as  $\text{low}^2$ , **fairly\_low** as  $\text{low}^{1/2}$ , and **not\_fairly\_low** as  $(-\text{low})^{1/2}$

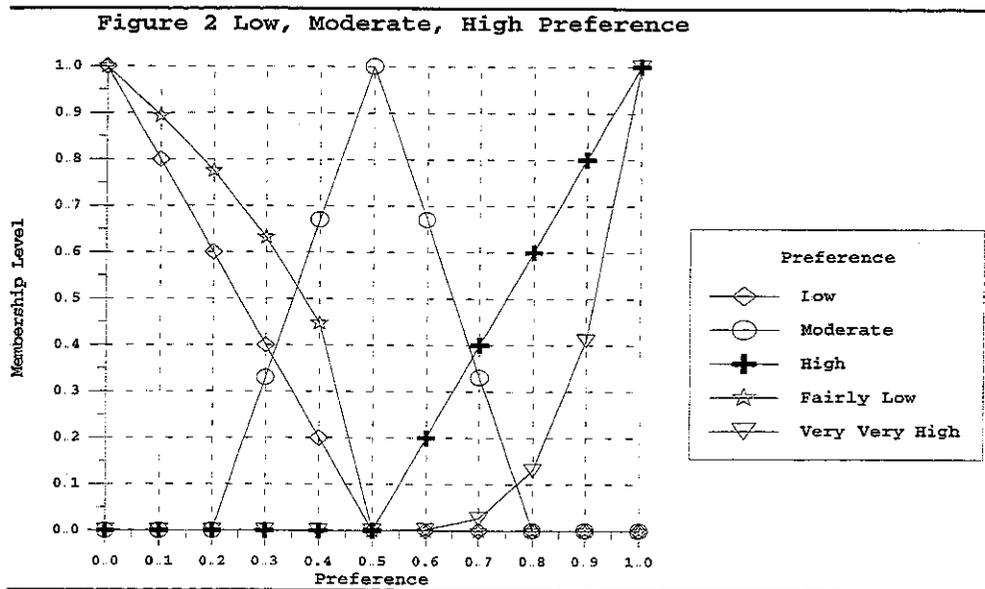
$U$  is a linguistic variable denoting preference defined on discrete base set  $Y = \{0.0, 0.1, \dots, 1.0\}$ . High, moderate, and low preference are primary linguistic values defined as fuzzy subsets of  $Y$ , respectively as

$$\text{high} = \{0/0, 0/0.1, 0/0.2, 0/0.3, 0/0.4, 0/0.5, 0.2/0.6, 0.4/0.7, 0.6/0.8, 0.8/0.9, 1.0/1.0\}$$

moderate = {0.0, 0.1, 0.2, 0.3, 0.33, 0.3, 0.67, 0.4, 1.0, 0.5, 0.67, 0.6, 0.33, 0.7, 0.0, 0.8, 0.0, 0.9, 0.0, 1.0}

low = {1.0, 0.8, 0.1, 0.6, 0.2, 0.4, 0.3, 0.2, 0.1, 0.4, 0.0, 0.5, 0.1, 0.6, 0.0, 0.7, 0.0, 0.8, 0.0, 0.9, 0.1, 0}

Very\_high is defined as high<sup>2</sup>, very\_very\_high as high<sup>4</sup>, and fairly\_low as low<sup>1/2</sup>. These are fuzzy subsets are illustrated in Figure 2.



For illustration, weights for antecedents are assumed to be the same within each rule, that is,  $w_{ij} = w_j$  for all  $i = 1, 2, 3, 4$ . Weights may be elicited from decision-makers using the pairwise comparison method (Saaty, 1980), though in this example, weight sets have been assumed to illustrate the potential of the proposed method

The calculation of the fuzzy subsets of projects associated with rule  $i$ ,  $A_i$  ( $i = 1, 2, 3, 4$ ), where  $A_i = \cap_{j=1}^6 A_{ij}$  ( $i = 1, 2, 3$ ) and  $A_4 = A_{42} \cup A_{43}$  are as follows

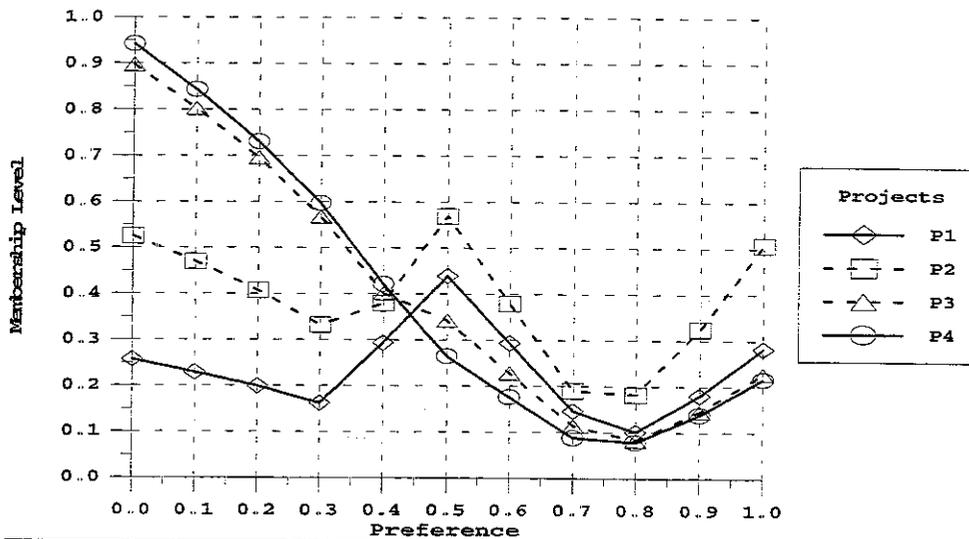
$$\begin{aligned}
 A_1 &= \{0.28P_1, 0.51P_2, 0.23P_3, 0.22P_4\} \\
 A_2 &= \{0.12P_1, 0.29P_2, 0.07P_3, 0.05P_4\} \\
 A_3 &= \{0.44P_1, 0.57P_2, 0.34P_3, 0.26P_4\} \\
 A_4 &= \{0.26P_1, 0.53P_2, 0.90P_3, 0.94P_4\}
 \end{aligned}$$

Thus,  $\tau_1(P_1) = 0.28$ ,  $\tau_1(P_2) = 0.51$ ,  $\tau_1(P_3) = 0.23$ ,  $\tau_1(P_4) = 0.22$ ,  $\tau_2(P_1) = 0.12$  etc. The rules may be represented more compactly as

- If  $A_1$  then  $B_1$
- If  $A_2$  then  $B_2$
- If  $A_3$  then  $B_3$
- If  $A_4$  then  $B_4$

where  $B_1 = \text{very\_high}$ ,  $B_2 = \text{very\_very\_high}$ ,  $B_3 = \text{moderate}$ , and  $B_4 = \text{fairly\_low}$ . Then the fuzzy subsets of  $P \times Y$ ,  $B_{0i}$  are aggregated as  $B_0 = \cup_{i=1,4} B_{0i}$  where  $B_0(p,y) = \vee_{i=1,4} B_{0i}(p,y)$ . The fuzzy subsets  $B_0(p,y)$ ,  $p \in P$ ,  $y \in Y$  are illustrated in Figure 3. Thus  $P_1$ , an

Figure 3 Preference for Projects



environmentally sensitive project, is the 'best' project. The results are  $COA(P_1) = 0.49$ ,  $COA(P_2) = 0.46$ ,  $COA(P_3) = 0.30$  and  $COA(P_4) = 0.29$ . Furthermore, results of an additive fuzzy system yield an equivalent ranking of projects. A hypothetical 'anti-ideal' project,  $P_*$ , which fails to satisfy any of the impacts/factors would not fire rules 1-3 but rule 4 would be completely satisfied with  $COG(P_*) = 0.164$ , whilst a hypothetical 'ideal' project,  $P^*$ , would completely fire rules  $i = 1, 2, 3$  but rule  $i = 4$  would not be fired. Thus  $COG(P^*) = 0.673$ . An index of performance based on  $COG(P_*)$  and  $COG(P^*)$  is  $V(P_i) = 100(COG(P_i) - COG(P_*)) / (COG(P^*) - COG(P_*))$ , yielding  $V(P_1) = 63.1$ ,  $V(P_2) = 58.6$ ,  $V(P_3) = 27.2$ , and  $V(P_4) = 23.8$  percent.

The above approach has assumed that the weights of antecedents are equal. When differential weights are introduced, the preference for options changes. For example, consider the weights  $W^{eng/econ} = \{w_1, w_2, w_3, w_4, w_5, w_6\} = \{2.75, 0.125, 0.125, 0.125, 0.125, 2.75\}$  emphasising non-environmental or engineering/economic factors (travel-time

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savings, capital cost) at the expense of environmental criteria and  $W^{env} = \{w_1, w_2, w_3, w_4, w_5, w_6\} = \{0.5, 1.25, 1.25, 1.25, 1.25, 0.5\}$  emphasising environmental factors at the expense of engineering/economic factors. In the former case,  $COA(P_1) = 0.34$ ,  $COA(P_2) = 0.45$ ,  $COA(P_3) = 0.28$ , and  $COA(P_4) = 0.45$ ; that is  $P_4$  and  $P_2$  are joint 'best'. In the latter case,  $COA(P_1) = 0.53$ ,  $COA(P_2) = 0.47$ ,  $COA(P_3) = 0.31$ , and  $COA(P_4) = 0.25$ ; that is,  $P_1$ , an environmentally sensitive project is 'best'. Though the above weights sets are extreme, they do illustrate the potential for differentially weighting antecedents. In practice, more realistic weight sets could be evolved and their implications for the identification of a 'best' project assessed.

### Conclusion

An application of fuzzy systems in the context of the evaluation of transport projects has been considered involving a rule-based fuzzy system. Projects are characterised in terms of multiple environmental impacts/factors. The method incorporates 'if...then' rules involving some or all of the (quantitative or qualitative) impacts/factors as antecedents and a level of preference as the consequent.

The method requires that performance of projects with respect to impacts/factors be assessed relative to each other which may be estimated either subjectively through pairwise comparison methods, or more objectively, through appropriate transformation of quantitative data.

The advantage of fuzzy logic-based systems includes a flexible framework within which to include soft impacts/factors. Obviously the mathematical form of the implication, the form of the connective 'and' and 'or' between antecedents, the form of the connective 'else' in the aggregation of 'if...then' rules, the choice of defuzzification method, *etc* all play an important part in the quality of the fuzzy model. Clearly much more research is required to assess the implications and merits of different possible structures and the practical value of fuzzy systems as a basis for the environmental evaluation of alternative transport projects.

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