Abstract:

Transportation road networks play an important role in regional economies, whether such regions are core dominated or polycentric. For sustained and rapid growth, regional planners must be able to estimate the supply of infrastructure in terms of new roads as well as improving existing networks. One of the key issues that arises in reaching such an estimate is that of determining how well connected an existing road network is within a region. The users of such a road network should be able to reach any part of the network from any other part to accomplish economic and social functions. Since regions and road networks do not generally evolve according to a fixed set of rules, it is difficult to measure the connectivity of a region and to compare it with other regions whose economies perform better or worse than the region under consideration. The goal of this paper is to develop and apply an analytical approach that can be used to build a connectivity index of road networks. The approach is based on fractal geometry and stream/river network analysis from the field of hydrology. It is shown that the road networks can be characterised by a fractal dimension, whose value indicates the denseness or sparseness of the road network in a region. Such an index could serve as an important policy tool for regional decision makers.

Contact Author:

Roger Stough
Institute of Public Policy MSN 3C6
George Mason University
4400 University Drive
Fairfax VA 22030 USA

telephone: +1 703 993 2280
email: rstough@osfl.gmu.edu

Rajendra G Kulkarni
Center for Regional Analysis
Institute of Public Policy
George Mason University
Introduction

The role played by infrastructure growth in the economies of core-dominated regions has been known for sometime (Talley, 1996; Hartmuck, 1996; Button, 1995; Waugh, 1993; Duffy-Deno and Eberts, 1991; Da Silva, Elson and Martin, 1987). However, many regions in the U.S. that were once dominated by a single center, have evolved into multi-center edge city networked regions that carry out competing yet at the same time compatible economic activities within these regions (Stough, Haynes and Campbell, 1997; Batten, 1995; Garreau, 1991). Spatial interaction models have used transportation costs as the basis for analyses of economic activities in regions (Paelinck, Ancot and Kuiper, 1983; Paelinck, Nijkamp, 1975; Klassen, Paelinck, Wagenaar, 1979). With the rise of edge-city dominated regions, mobility and ease of flow among different centers in a region becomes more important. Hence, how well connected a network of edge cities is, becomes a prime concern of policy makers in polycentered regions. But how does one measure connectivity, either for a single core-dominated region or in particular, for multi-center edge city regions?

The traditional graph theoretic measures based on gross characteristics (Cyclomatic numbers, Beta index, Alpha index, Gamma index) and the shortest-path characteristics (diameter, accessibility index, dispersion index of networks) (Haget, Chorley, 1969; Kansky, 1963), are complex and prone to the combinatorial explosion. Hence an approach that would overcome the dual problems of simplistic or trivial measures and combinatorial explosion would be useful. Decision makers would benefit if a method for assessing road network connectivity was available. Among other things it could be used to evaluate the relative quality of road infrastructure.

In this paper we develop an analytical approach that produces an index that can be used to assess the connectivity of regional road networks. Road network connectivity is defined in two ways: global connectivity ‘G’ and nodal connectivity ‘C’.

1. Global connectivity measures how accessible any part of the network is from any other part of the network. It does not take into consideration which nodes among many are important and which are not. The global property of connectivity - an intuitive concept of connectedness of a network - is quantified as a global (macro) connectivity index ‘G’.

2. Nodal Connectivity measures how well connected major nodes in a network are to the rest of the network. Thus, each major node may have a different connectivity index ‘Cj’, where ‘j’ refers to the node under consideration and may in turn contribute to the global connectivity ‘G’, where ‘G’ is computed as an average or weighted average of all the nodal connectivity indices of a region. Note that the nodal connectivity refers to a meso-level connectivity rather than the micro level connectivity associated with the connectedness of individual housing units or employment firms. The issue of the accessibility of each residential unit to every other point in the network is not addressed in this paper and is left
Regional Road Networks and Connectivity

to be addressed in a different framework than the one developed here. Therefore further consideration of micro-level connectivity is dropped for the remainder of this paper. Micro-level connectivity involves modeling each user's perception of accessing different parts of road networks, because each user does not use every part of the network but has a need to travel to only selected areas for routine activities. Similarly, meso-level connectivity deals with parts of entire networks and is not considered in the context of nodal connectivity. At the same time, we must point out that the analytical framework developed in this paper would allow us to compute a connectivity index for a small area or any part of the entire regional network.

Both global and nodal connectivity are examined using the same analytical approach in this paper. The approach is based on fractal geometry (Mandelbrot, 1982) and the methodology is conceptually similar to the one used to analyze river and stream networks (Strahler, 1952; Horton, 1945). However, road networks are not the same as river and stream networks. An important and obvious difference is that unlike river networks which are unidirectional, traffic on road networks flows up- and downstream as well as across the system in a web-like fashion. In fact, web-like road networks have multiple directional flows and circuits (closed paths that have flow starting from and ending at the same locations). Thus, any approach which is derived from river network analysis must be adjusted to address the web-like nature of road networks.

Although we have stressed the differences between river and stream networks and road networks, there is one fundamental property that is common to both. A stream network drains all of the water in an area. Similarly, a road network in an area must provide roads that can in theory allow people in the area to access every part of that area. In other words, a road network must have, in principle, the capacity to "drain" all users of an area just like stream networks drain water.

Road traffic networks

A regional traffic network may consist of the following:
1. multiple major employment/business centers (such as edge cities),
2. scattered and/or clustered residential areas, and
3. small employment/business locations (such as strip malls, shopping centers)

Next, we briefly discuss how employment centers and residential areas are located in a network and what their demands on the network are in terms of connectivity to the rest of the network. Residential areas may be viewed as located in the intervening spaces between employment centers and along smaller roads and cul-de-sacs, although in some areas they may be mixed in with employment centers. While employment centers are typically located at the intersections of major roads and adjoining areas. Residential locations have finite dimensions and hence are not points but areas that consist of individual households. In theory, each such residential cluster can be represented as a separate node identified by a centroid. Similarly strip-malls, shopping centers as well as independent employment
centers constitute employment locations or employment nodes. Both the residential nodes and the employment nodes need to be connected to other similar nodes, to each other and to the major employment centers to satisfy the demands of commerce and living.

Daily demands for accessibility to the road network from residential nodes are quite different in strength and nature from those of employment nodes. Whether every node of the network needs to reach every other node or not, a basic road network infrastructure must exist to take into account such anticipatory demands and thus provide necessary roads for traffic movement. The demands put on the road network to reach the major employment centers are even greater, from both residential and employment nodes. Usually there is a steady or regular demand for accessibility to residential and employment nodes among themselves, while the demand on access to major employment centers is a function of the time of the day. Thus an analytical approach must take into account a vast number of nodes and locations and with different levels of demand for accessibility. So, how can the vast number of nodes and their connecting links be handled without getting bogged down in a combinatorial nightmare? In the following sections a combination of fractals and stream networks is presented to help assess the connectivity of traffic networks.

Methodology

Fractal nature of road networks

The fractal approach proposed in this paper, uses number of lanes on road segments to develop measures of the properties of road network connectivity. The road networks are seen as irregular fractal objects (see Figure 1) that are statistically self-affine. An object is considered as being self-affine as opposed to being self-similar if it maintains its appearance under differential changes in the scale of its dimensions (Barabasi, Stanley, 1995; Bunde, Havlin, 1991; Mandelbrot, 1982). Thus regular shaped objects are usually self-similar while irregular objects may be self-affine. It will be shown that the apparently

Figure 1 Road Network as a self-affine fractal structure
The random nature of road networks can be described by scaling laws, whose exponents are used to develop the fractal dimension of these networks (Mandelbrot, 1982). We will also utilize the conceptual framework as outlined in (Barbera, Rosso, 1989) and (Nikora, 1991) and apply these concepts to regional road networks. The field of hydrology has produced a considerable body of literature on the fractal nature of river, stream or channel networks and their connectivity (Masek, Turcotte, 1994; Kirchner, 1993; Karlinger, Troutman, 1992; Stark, 1991; Shreve, 1965; Shreve, 1969), which we also draw upon in our analysis of road network connectivity.

An aerial view of regional road networks as shown in Figure 1, appears as a self-affine structure of a randomly branching network on a two dimensional Euclidean surface. The network is composed of roads and the smallest segment of a single lane road is a section that has a certain width and length, and hence it may be considered as a two dimensional ribbon-like Lebesgue measure (Karlinger, Trotman, 1992). The Lebesgue measure is a basic unit that can be used to count a certain property of an object by “filling up” that object with the basic unit. For example, if we wished to measure the number of lanes by segments on a long stretch of road, we could use a pre-defined Lebesgue measure consisting of a two dimensional piece of road that has a certain width and length. Also, it is worth noting that though networks of the type shown in Figures 1 may have circuits, these circuits can be eliminated by considering the links as merely self-touching or self-contacting (Mandelbrot, 1982) and not intersecting themselves.

A multiple number of this basic unit (the Lebesgue measure) then fills up that stretch of road completely. Unlike the Peano curve that fills the entire two dimensional plane but still has a dimension of 1, a branched road network has a fractal dimension that is more than 1 (due to the Lebesgue measure with a finite area) but less than 2 (since it does not fill the entire two dimensional Euclidean space). Of course, depending on the sparseness or denseness of the branching network, its fractal dimension would be nearer to 1 or 2, respectively. We use the computed fractal dimension of road networks to compare the property of connectivity among different networks. In other words, we assert that, when comparing two road networks, a network with a fractal dimension ‘d’ is less well connected than a road network that has fractal dimension ‘d + ’ and vice versa. We
hypothesize that a large deviation above the numerical value of 1 or below 2 may be an indication of either the lack of or an excess of built infrastructure at a particular time, or across time if data for multiple time periods is available.

**Fractal dimension of a network**

Perception of physical objects in terms of integer dimensionality is easily understood. Although unnatural as it sounds, nature is better described with the use of noninteger dimensions. But how does one measure the noninteger dimension? For example, the length of a coast line depends on the gauge used to measure it. As a finer gauge is used, the length of the coast does get longer (Mandelbrot, 1982). The nature and form of coast lines can be better understood in terms of scaling laws, which in turn explain the noninteger dimensionality of coast lines. The following figure shows the plot of the gauge used to measure the length and the length ‘L’. The length becomes a function of the gauge used (Mehaute, 1990). In other words,

![Figure 3 Network with three employment/business centers](image)

the length ‘L’ in terms of the gauge ‘e’, has the following relation:

\[ L \approx e^{-D} \]  

(1)

Which in turn is a function of the number of segments ‘N’, each of length ‘e’ and is given by:

\[ N(e) \times e = e^{1-D} \]  

(2)

Thus,

\[ N(e) \approx e^{-D} = \left(\frac{1}{e}\right)^D \]  

(3)

Then the ‘D’ can be computed as follows:
Any other measure such as change in width of roads (change in number of lanes). For each such segment we assign a number depending on the number of lanes on that segment. This number is called the ‘order’ of the segment. Thus, a road segment with one lane each way receives a code of 1, two lanes are assigned a value of 2 and so on. In other words the number of lanes of a section of a road becomes the order of that segment. A single lane road, used by both to and from traffic receives an order of 0.5 and the cul-de-sacs as well as dead-ends receive an order of ‘0’. Thus the order number measures the number of lanes on that segment (see Figure 3). In general, a ‘0’ segment merges or connects to a segment of order ‘0.5’ or ‘1’. The segment with orders ‘0.5’ and ‘1’ merge into segment with order of ‘2’ and so on. But unlike river networks where a segment of lower order can merge into a branch that has at least the same or higher order, the number of lanes on roads can vary and thus our coding scheme does not require that the merging of different segments must follow a strict rule as observed in case of river networks. This schema has the advantage that it allows for variation in the road segment lanes and takes care of lane width enlargement near junctions and traffic islands and due to other natural topological factors that force changes in the width of roads. Having assigned the ‘order’ number to each and every segment in the network, we compute the connectivity indices as follows. Since the global connectivity measures connectivity across a network rather than from a node to another node, the individual nodes can be ignored when global connectivity is computed. Assuming that the segment orders are, 6, 4, 3, 2, 1, 0.5, 0 and the number of each order segment is \( n_i \) where \( i \) is the order of these segments and \( N_i \) is the bifurcation ratio of the network. Similarly, we could find a ratio of the type given by:

\[
\frac{n_i}{n_{i+1}} = N_i ,
\]

(5)

where ‘\( i \)’ is the order of these segments and ‘\( N_i \)’ is the bifurcation ratio of the network. Using equations (1) through (4) we can combine equations (5) and (6) as follows:

\[
N_i \approx (L_s)^\Delta
\]

(7)

From which we can get the value of exponent ‘\( \Delta \)’ by taking log of both sides of equation (7), and obtain the following:

\[
\log(N_i) = \Delta \times \log(L_s)
\]

(8)

\[
\Delta = \frac{\log(N_i)}{\log(L_s)}
\]

(9)
Substituting for ‘N,’ and ‘L,’ in the above equation, we get,

\[ \Delta = \frac{\log(N_x)}{\log(L_x)} = \frac{\log(n_i)}{\log(l_{i+1})} \]  \hspace{1cm} (10)

Equation (13) can be expanded using the laws of logarithm into the following:

\[ \Delta = \frac{\log(N_x)}{\log(L_x)} = \frac{\log(n_i)}{\log(l_{i+1})} = \frac{\log(n_i) - \log(n_{i+1})}{\log(l_i) - \log(l_{i+1})} \]  \hspace{1cm} (11)

or alternately,

\[ \Delta = \frac{\log(N_x)}{\log(L_x)} = \frac{\log(n_i)}{\log(l_{i+1})} = \frac{\log(n_i) - \log(n_{i+1})}{(i+1-i) \log(l_i) - \log(l_{i+1})} \]  \hspace{1cm} (12)

The right hand side of equation (12) can be computed from the slopes of the semi-log plots in Figure 4. The left axis is the log of the count (frequency) of the segments of each order while the horizontal axis has the order of these segments (abscissa). The log of the number of segments is inversely proportional to the order of these segments. On the other hand, log of the average length of the segments of an order (right hand axis) is directly proportional to the order of that segment (abscissa). The fractal dimension ‘\(\Delta\),’ then is the global connectivity index ‘G’ for the network. If we have a denser road network (that is the bifurcation ratios are higher than the average length of the segments) then obviously, the value of ‘\(\Delta\)’ is higher as compared to a sparse road network.

To compute nodal connectivity, we have to consider one node at a time and follow the same procedure as we did for the global connectivity G. There will be little or no difference in the connectivity index if the network configuration does not change, that is, if
The infrastructure remains the same. But, in reality, there is always some change in the network configuration when the network is viewed one node at a time. First, some of the links leading to other nodes, for example, nodes 'B' and 'C', (Figure 3) can be neglected in the computation of connectivity (nodal) of node 'A' (see Figure 5). Secondly, one may assign a different "order" to each segment depending on the real demand to travel on that node, whereby the "order" becomes the "effective order" of the segment that reflects a combination of factors such as capacity, number of lanes, actual traffic flow, the time of the day and any other relevant information. The nodal connectivity index is computed just like the global connectivity index

$$\frac{n_i}{n_{i+1}} = \frac{N_b}{N_b}$$

(13)

where $N_b$ refers to the bifurcation ratio and

$$\frac{L_i}{L_{i-1}} = L_b$$

(14)

followed by,

$$\Delta_n = \frac{\log(N_b)}{\log(L_b)}$$

(15)

where the fractal dimension $\Delta_n = C$, the nodal connectivity index. In a region that has 'm' major nodes, one may compute the global connectivity index 'G' from the nodal connectivity indices for 'm' nodes as follows:

$$G = \sum_{i=1}^{m} C_i$$

(16)
Preliminary results of Global Connectivity Index

Below we give the preliminary results of the analysis carried out on two independent jurisdictions in Virginia, Stafford county and Fredericksburg city. The new definition of the Washington-Baltimore CMSA in the U.S. includes Stafford county, while the city of Fredericksburg is at the southern tip of the I-95 freeway in Stafford county. Stafford county and Fredericksburg city represent the self-affine road networks defined in the earlier sections. Both have I-95 running along their lengths and state highways along their southern borders. Residential and commercial areas have developed along both sides of these main roads. The computation of the fractal dimension computation was carried out by building the area road network maps from the U.S. Bureau of Census TIGER/Line data files (1991) using the Census Feature Class Code (CFCC) for various types of roads. Figure 6 shows the area road network map for Stafford county (VA) while Figure 7 shows Fredericksburg city (VA). In both cases, Interstate I-95 is a major backbone that runs along a north-south axis. Nearly all other types of roads such as state, county and city roads either feed into or branch out from this main trunk. The plots in Figure 7 and 8 show the computation of the bifurcation ratio and mean length for each of these road networks. Note that the TIGER/Line data files provide only the length and default identification/annotation data for each segment of the area road networks. The important and desirable attributes such as the lanes per segment of road and road capacity per segment are not provided by these data files. The Stafford county plot (Figure 8) shows three types of roads, namely, (i) Primary Highways with limited and unlimited access, (ii) Secondary and Connecting Roads and Connecting Roadways and (iii) Local,
Neighborhood and Rural Roads. While, the Fredricksburg city road network is identified with four types of roads, namely, (i) Primary Highways with limited access (ii) Primary Highways with unlimited access, (iii) Secondary and Connecting Roads and Connecting Roadways and (iv) Local, Neighborhood and Rural Roads. The fractal dimensions were computed using equations (8) and (9) for both, the Stafford and Fredricksburg road networks. Thus for Stafford county, From Figure 8, we have the following values: The bifurcation ratio $N_s = 15785$ and the mean length ratio $L_s = 0.0096$

$$\Delta_{\text{stafford}} = \frac{\log(15785)}{\log(L_s)} = \frac{319845}{2017729} = 0.0096$$

Similarly, from Figure 9, we get the following for Fredricksburg city: The bifurcation ratio $N_f = 231$ and the mean length ratio $L_s = 0.00221$.

$$\Delta_{\text{fredricksburg}} = \frac{\log(2318)}{\log(L_s)} = \frac{2315113}{1655608} = 1428547 \approx 14$$

The city has lower connectivity index than the county.

**Figure 8 Stafford county, VA road network**: Road types given by the CFCC code (i) Primary Highways with limited and unlimited access, A10-A29 (ii) Secondary and Connecting Roads and Connecting Roadways A30-A39 and (iii) Local, Neighborhood and Rural Roads A40-A49, against the count of segments and mean distance.

**Figure 9 Fredricksburg city, VA road network**: Road types: CFCC code (i) Primary Highways with limited and unlimited access, A10-A19 (ii) Primary Highways with unlimited access, A20-A29 (iii) Secondary and Connecting Roads and Connecting Roadways A30-A39 and (iii) Local, Neighborhood and Rural Roads A40-A49, against count of segments and mean distance.
Conclusions

The fractal dimensions of road networks serve as connectivity measures for these networks. Both global and nodal connectivity indices can be used to measure connectivity regardless of the network of shapes and sizes. Thus various networks can be easily compared to each other using the connectivity measures. The connectivity indices can also be computed for sub-parts of a network and used to compare these subparts to assess the differential demand of or lack of infrastructure. The connectivity models based on the more traditional approach of graph theory are not capable of obtaining this type of information, especially when comparing a multitude of road networks of different sizes. They provide even less information to policy makers for assessing relative adequacy of the built infrastructure of a region. At the same time, it should be noted that the approach in this paper neither supersedes nor replaces the traditional graph theoretic approaches. Instead, the aim is to make available to policy makers a method that can give them information about area road networks which would otherwise be difficult to achieve from traditional models and measures.

The method proposed in this paper is both intuitive and empirical. It is easy to implement at all scales of network sizes. Almost without exception transportation authorities in all regions have access to information on lanes per segment of regional roads as well as the lane miles and areas of these networks. Hence, computation of the connectivity index becomes a simple exercise of measuring and rank ordering the lanes, lane miles and calculating fractal dimensions.

References


Paelinck, J H P and Nijkamp, P (1975) *Operational Theory and Method in Regional Economics*. Saxon House and Lexinton, Massachusetts


TIGER/Line™ Files (1991), Census Feature Class Codes, Appendix E, TIGER/Line Census Files, 1990 prepared by the Bureau of the Census, Wash D C., The Bureau