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Efficient Congestion Tolls in the Presence of Unpriced Congestion: A Case with Non-identical Road Users

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Abstract

A deterministic user-equilibrium route choice model is used to simulate the behaviour of drivers in response to toll charges. The objective of the simulation is to derive the socially optimal tolls in the context of non-identical road users, where drivers in different income classes are assumed to have different trade-off between paying a price versus saving a minute of travel time. The non-identical user treatment is a departure from traditional analysis, which treats road users as identical. Welfare maximising tolls were determined under three pricing regimes: first-best (when all roads can be subject to pricing), second-best (when only portion of the network can be priced), and no pricing. Two types of network were modelled; the simple two-route network and the general network case. The simulations reveal significantly different traffic diversions, and consequently different optimal toll patterns in the case of non-identical road users. In terms of welfare implications, the results indicate that the lower-income groups are the most affected by pricing. When all routes are subject to marginal cost pricing, low income drivers are more worst off. On the other hand, the higher income groups are the likely winners, particularly when pricing is only applied on portion of the urban road network.

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Introduction

Recent technological advances and the growing problem of urban road congestion are the main impetus behind the renewed interest in road pricing. The world's first fully automated congestion pricing system in California (the State Route 91 express lanes in Orange County) was opened in December 1995. One of the important features of the SR-91 facility is that the two priced lanes (in each direction) are competing with the existing four lanes which are toll free (Sullivan, 1998). The successful implementation of SR-91 has gained some acceptance from the public for some form of pricing. Thus the focus of pricing and feasibility studies has shifted to limited pricing.

The case of limited pricing, where alternative free routes exist, raises two important questions. The first is how much to charge in order to achieve an optimal allocation of resources. The second question is who wins and who losses under this limited form of pricing, i.e. the welfare implications. The answer to the first question is found in the earlier studies of Levy-Lambert (1968) and Marchand (1968) who derived the efficient toll for the simple two-route network. Some recent studies include Arnott *et al* (1990), McDonald (1995), Verhoef *et al* (1996), and Liu and McDonald (1998). Liu and McDonald derived the second-best efficient toll considering both the peak and the off-peak periods. Verhoef (1998) attempted to provide a theoretical framework to derive the second-best optimal toll for the case of more than two routes, i.e. the general network case.

One important issue that has not been addressed in the above studies, and is recently receiving attention, is the differing cost-time trade-off among road users. Previous studies have adopted the traditional approach that all users have identical perception of their travel cost. However, in the presence of pricing, this simplifying assumption is inconsistent with actual behaviour. Individuals do differ in their willingness to pay a price in order to save travel time.

In an earlier paper (Sapkota, 1998), the author undertook a simulation study to predict the route choice responses of non-identical users to a fixed toll charge. Users were assumed to vary in their perception of the charge relative to their income. The choice of income segmentation was based on the assumption that travel time valuation is highly correlated with income (Anderson and Mohring, 1996). The route choice experiment under a road pricing system was simulated using both the simple two-route network model and a general network case. Results of the simulations indicated a different allocation or distribution of demand in the case of non-identical users. Most importantly, the results showed better prediction of demand for the tolled road

In this paper, the route choice analysis with non-identical users is extended to deal with the optimal pricing problem. The aims of the simulations are twofold. The first is to derive the efficient second-best tolls. The second is to examine the welfare implications of the second-best tolls. For comparison purposes, two other pricing regimes (first-best pricing and no pricing) are modelled. The route choice experiments are carried out in both the simple and the general network cases

Framework for the optimal pricing problem with multiclass users

Calculation of optimal tolls

The framework for welfare maximising optimal tolls for the two-route network model is adopted from Verhoef *et al* (1996). Two routes, a motorway and an arterial road, connecting an origin and a destination are considered In the ideal case that both routes can be priced efficiently, the first-best optimum price for each link, f_a, is obtained using the expression:

$$f_a = N_a \; c'_a(N_a)\,, \qquad a = A, M \tag{1}$$

where the subscript *a* refers to the priced links (A the arterial route and M the motorway route). N_a is the total demand on link *a*, and $c'_a(N_a)$ is the slope of the average cost curve of link *a*. Note that Equation (1) applies to the general network case

The second-best efficient toll is derived under the constraint that only one route can be priced. In this case, only the motorway is tolled. The one-route toll still maximises society's welfare. This efficient toll, f, is determined from the expression:

$$f = N_M \cdot c'_M(N_M) - N_A \cdot c'_A(N_A) \left(\frac{-D'(N)}{c'_A(N_A) - D'(N)} \right) \tag{2}$$

where N_M is the total demand on the toll route (Motorway), N_A is the total demand on the untolled route (Arterial Road), $N = N_M + N_A$, $c'_A(N_A)$ is the slope of the average cost curve of the toll route, $c'_M(N_M)$ is the slope of the average cost curve of the untolled route, and D'(N) is the slope of the demand curve for total trips

The first term in (2) is the marginal external congestion costs on the toll route in the second-best case. The second term indicates that, for optimal acceptance of the fee, only a fraction of the marginal external costs on the untolled route should be charged to take into account the "spill-over" effects from the tolled route. However, in the case of perfectly inelastic overall demand, the expression inside the large parentheses in (2) approaches unity. Thus, (2) reduces to

$$f = N_M \cdot c'_M(N_M) - N_A \cdot c'_A(N_A) \tag{3}$$

In the present model, a perfectly inelastic total demand is assumed and the two routes are perfect substitutes. Therefore, (3) is used in the simulation that only takes into account the effect of route switching as the socially optimal toll is imposed to improve efficiency in utilisation of the available capacity.

Multiclass user equilibrium assignment

A simplified deterministic multiclass user equilibrium approach, used earlier in Florian (1998), has been adopted.

Assume the demand between O-D pair w can be divided into k income classes. If the generalised travel cost comprises of only the travel time and the toll charge, then the cost of link $\boldsymbol{a}$ perceived by a user of class k can be written as

$$s_a^k(v_a) = s_a(v_a) + \theta^k P_a^k \ , \quad v_a = \sum_{k \in K} v_a^k \ , \ a \,\varepsilon A \tag{4}$$

where, $a \,\varepsilon A$ is the set of links, $s_a(v_a)$ is the travel time cost for link a at flow v_a, θ^k is the value of time for each class k (expressed in time/unit of cost), P_a^k is the toll price imposed on link a for each class k. Under optimal pricing, $P_a^k = f_a^k$ for first-best toll, and $P_a^k = f^k$ for second-best toll.

The cost of a path for each user class with a single tolled link is given as

$$s_r^k(v) = \sum_{a \in A} \delta_{ar} s_a(v_a) + \delta_{ar} \theta^k P_a^k \qquad r \in R_w^k \ , w \,\varepsilon W, k \,\varepsilon K \tag{5}$$

Where $r \in R_w^k$ is the set of routes connecting O-D pair w for user class k, $w \,\varepsilon W$ is the set of O-D pairs, $k \,\varepsilon K$ is the set of user classes, and $\delta_{ar} = 1$ if link a belongs to path r and zero otherwise.

The link cost function given in (4), expressed in generalised time units, implies that the different classes are subject to the same congestion effect. However, each class perceives a different constant bias. This perceived constant bias is reflected by the varying values of time.

As discussed in Florian (1998), the simplified multiclass user equilibrium model is equivalent to the classic deterministic network equilibrium model that satisfies the Wardrop (1952) user equilibrium condition. In addition, the simplified multiclass user equilibrium model is a convex cost minimisation problem, which has a unique optimum solution. The numerical solution of this model by the linear approximation method can be found in INRO Consultants Inc (1998). For each O-D pair w, at user equilibrium, no user in each class can improve his/her travel time by unilaterally switching routes.

Optimal pricing with multiclass users: two-route network case

The simulation model

Like many other studies dealing with the two-route network case (Marchand, 1968; Arnott *et al* 1990; McDonald, 1995; Braid, 1996; Verhoef *et al* 1996; Liu and McDonald, 1998), the present simulation study considers a simple urban highway system, and deals the problem of the morning commute. The routes are considered perfect substitutes, but one route is superior to the other as it has higher capacity and lower free-flow travel cost Attributes of the simple network and the parameters of the supply function are given in Table 1. The functional form of the travel time function given in (6) is of the BPR type (Branston, 1976) Equation (6) is used in the supply function in (4). The simulation is designed for the one-hour morning peak period.

$$s_a(v_a) = t_a(1+\alpha(v_a / Q_a)^{\beta}) \ , \ v_a = \sum_{k\in K} v_a^k \qquad (6)$$

where for link a, $s_a(v_a)$ is the travel time at flow v_a (min), t_a is the free flow travel time (min), v_a is the vehicle flows summed for all income classes (veh/hr), Q_a is the link capacity (veh/hr), α and β are parameters Note that the effect of the vehicle operating cost is assumed not significant

Table 1 Attributes of the simple network model

	Arterial Route	Motorway
Capacity (vph)	1500	3000
Distance (km)	18	16
Speed (kph)	60	80
α	0.6	0.6
β	3.0	3.0

A fixed number of regular commuters are assumed, implying an inelastic overall demand. The overall demand is made up of commuters with different income levels, 12 income classes considered in the analysis, as shown in Table 2. However, for ease in comparing the effects of pricing, the 12 income classes are aggregated into 3 income groups; low, medium, and high. The proportion of trips by income groups in Table 2 are partly based on the congestion pricing study for the Twin Cities Metropolitan Area by Anderson and Mohring (1996) The demand combination with 20% low, 40% medium, and 40% high-income commuters is based on the 5-income groups used in the Twin Cities study. Furthermore, the values of time by income classes are derived using a similar approach to that of Anderson and Mohring. The average value of time for all users, ie with identical users, is assumed at $10.00 per hour.

To test the sensitivity of the optimal tolls under non-identical user assumption, four different demand combinations were modelled. This could be interpreted to represent low, medium, medium to high and high-income corridors or suburbs.

Table 2 **Travel demand data for the multiple income class assignment**

	Income Range	Average Income	Average Value of Time(1) ($/hr)	Demand 50/30/20(2)		Demand 35/35/30(2)		Demand 20/40/40(2)		Demand 10/20/70(2)	
				%Trips	#Trips	%Trips	#Trips	%Trips	#Trips	%Trips	#Trips
Low	<$20,000	$15,000	$2.00	5%	150	4%	120	3%	90	1.5%	45
	$20,000-$25,000	$22,500	$3.60	12%	360	8%	240	5%	150	2.0%	60
	$25,000-$30,00	$27,500	$5.40	15%	450	10%	300	5%	150	3.0%	90
	$30,000-$35,000	$32,500	$7.55	18%	540	13%	390	7%	210	3.5%	105
Medium	$35,000-$40,000	$37,500	$9.40	8%	240	8%	240	10%	300	5.0%	150
	$40,000-$45,000	$42,500	$10.60	8%	240	9%	270	10%	300	5.0%	150
	$45,000-$50,000	$47,500	$11.90	8%	240	9%	270	10%	300	5.0%	150
	$50,000-$55,000	$52,500	$13.10	6%	180	9%	270	10%	300	5.0%	150
High	$55,000-$60,000	$57,500	$14.40	4%	120	6%	180	5%	150	15.0%	450
	$60,000-$65,000	$62,500	$15.65	4%	120	6%	180	8%	240	15.0%	450
	$65,000-$70,000	$67,500	$16.90	4%	120	8%	240	8%	240	20.0%	600
	>$70,000	$80,000	$20.00	8%	240	10%	300	19%	570	20.0%	600
Total				**100%**	**3,000**	**100%**	**3,000**	**100%**	**3,000**	**100%**	**3,000**

(1) Average value of time for identical users, ie. irrespective of income level = $10.00 per hour.

(2) Demand 50/30/20: Income group with 50% low, 30% medium, and 20% high.
Demand 35/35/30: Income group with 35% low, 35% medium, and 30% high.
Demand 20/40/40: Income group with 20% low, 40% medium, and 40% high.
Demand 10/20/70: Income group with 10% low, 20% medium, and 70% high.

The simulations include three pricing regimes; no pricing, first-best pricing (both routes are priced), and second-best pricing (only the motorway is priced). Table 3 summarised the various cases modelled.

Table 3 Cases modelled

Demand Mix	Identical Users			Non-identical Users		
	NP	FBP	SBP	NP	FBP	SBP
Demand 50/30/20	✓	✓	✓	✓	✓	✓
Demand 35/35/30	✓	✓	✓	✓	✓	✓
Demand 20/40/40	✓	✓	✓	✓	✓	✓
Demand 10/20/70	✓	✓	✓	✓	✓	✓

Model results

Equilibrium flows and optimal fees

Table A and Figure 1 present the results of the cases modelled. In the absence of pricing, the corridor capacity is inefficiently utilised. The motorway is operating at about 94% of its capacity while the arterial route is underutilised at 12%. With the price mechanism in place, both routes are efficiently utilised. In terms of demand allocation, the equilibrium flows achieved under first-best and second-best pricing regimes are the same simply because the second-best price is set to maintain the optimal utilisation of both routes.

The non-identical user assumption appears to have significant effect in the resulting equilibrium flows, particularly under second-best pricing. The greater proportion of low-income users resulted in higher diversions to the arterial route. Conversely, the greater proportion of high-income commuters resulted in lower diversions to the unpriced alternative. Consequently, the optimal tolls obtained under the two specifications appear to be significantly different. With identical users, the same optimal tolls are required irrespective of the demand mix. On the other hand, different optimal tolls are required for the non-identical case. Furthermore, a much lower second-best toll is required when low-income commuters largely dominate the demand.

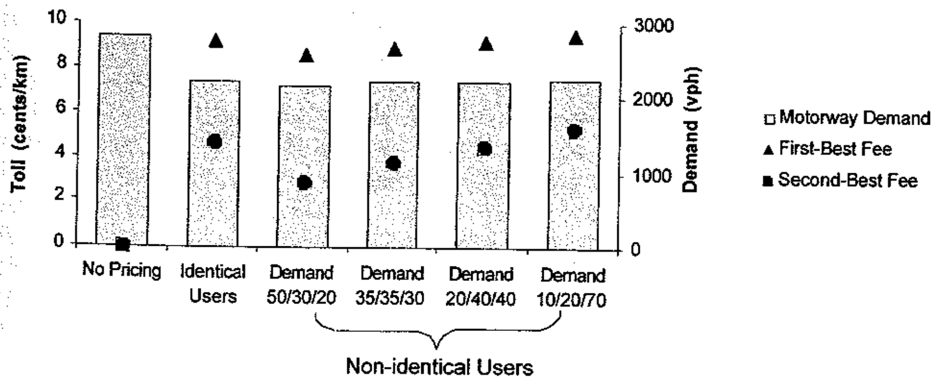
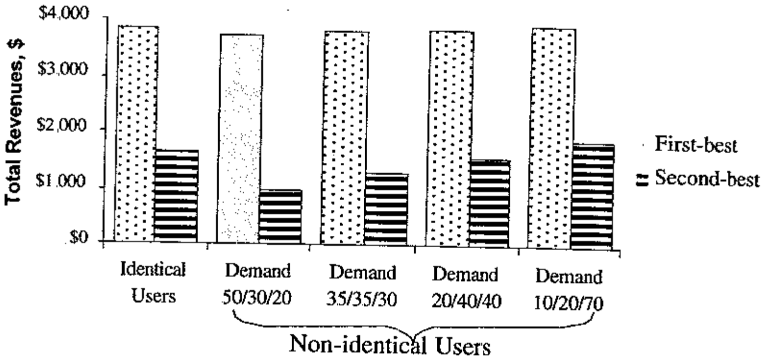


Figure 1 Demand and optimal tolls on the Motorway

Total Revenues

Figure 2 shows the revenues generated under first-best and second-best pricing. As expected, the constraint of pricing only one route at the optimal flow will result in lower revenues. Under first-best, the total revenues appear to be unaffected by the assumption of user heterogeneity (ie. non-identical users) In contrast, user heterogeneity appear to have significant effect on the revenues under second-best At the extreme, a much lower revenues relative to the identical case is generated in the case dominated by low-income users, while higher revenues is achieved in the case dominated by high-income users.

Figure 2 Revenues from optimal pricing



Total social and private costs

A summary of the total social and private costs is given in Table B. In the absence of pricing, the total social costs of travel far exceeded the total costs borne by road users themselves. The high societal cost accounts for the large externality cost of congestion resulting from the inefficient use of the network. Thus, pricing both routes at the marginal social cost is necessary to eliminate this social cost. On the other hand, the constraint of not pricing one route will result in some loss to society. The extent of net loss to society as shown in Table B indicates that the identical user assumption would be less accurate in reflecting the benefit from optimal pricing.

Total private costs by income groups

As mentioned earlier, the 12 income classes are aggregated into low, medium and high-income groups (see Table 2). The total private costs aggregated for each of the three income groups are summarised in Table C When both routes are optimally priced, commuters' private costs increased dramatically, about 22% for identical users (for all income groups) and up to 34% for low-income users under the non-identical assumption. Thus, first-best pricing is a very unpopular transport policy, especially because its effect is regressive Under second-best, the increase in private costs is less than a third of the first-best in the case of identical users A quite different outcome is revealed under non-identical users. It appears that at second-best pricing some groups actually incur lower travel costs In all cases, low-income commuters would tend to be real losers.

Optimal pricing with multiclass users: general network case

The simulation model

For the general network case, the road network covering the northern portion of the Perth urban area is considered. The network has 213 zones, 6000 nodes, and 2300 links. Due to lack of data, a synthetic demand is used. The synthetic demand matrix has 90,738 origin-destination trips. Only one demand combination, i.e. with income mix of 20% low, 40% medium and 40% high income groups (as per Table 2) is assumed for all zones. Furthermore, the overall demand is assumed inelastic. Thus, only the effects of routes switching are taken into account.

All the three pricing regimes were modelled for each user type. Under first-best pricing, each link of the road network is charged a fee equal to the difference in the marginal social and average cost of travel (also called the GAP) on each link. The average value of time for the identical user assignment is $10.00 per hour. The values of time for the non-identical case correspond to the 12 income classes in Table 2. Under second-best pricing, the southbound section of Mitchell Freeway is charged a fee equal to 30 percent of the GAP. Figure 4 shows the location of the tolled section. The choice of the second-best fee is simply arbitrary at this stage of the analysis. An approach to find the second-best optimum charge for the general network case is currently being investigated by the author. Anderson and Mohring (1996) used 25% of the GAP as the second-best charge for all expressways in their pricing study for the Twin Cities, where they used 5 income groups.

Model results

The simulation results are presented in Figures 3 and 4 as differences in predicted flows between the two user types. The darker shades indicate a prediction of increased trips under the non-identical assumption while the lighter shades indicate fewer trips. The two important results revealed in the first-best case are the significant increase in the total vehicle-hours, and the total vehicle-kilometres travelled by non-identical users. This seems to imply that low-income commuters are forced to travel on circuitous routes (perhaps incurring longer travel times), thus increasing rat-running on local streets. Under the second-best case, the interesting result is the prediction of increased traffic on the priced facility. The three northernmost links of Mitchell freeway shown in Figure 4 were relatively congested, therefore attracting higher tolls. Consequently, more trips are diverted to alternative routes under the assumption of identical users. When non-identical users are assumed, the model predicted more trips on the three northernmost links, while decreasing traffic downstream. The reason for the decrease is due to lower-income commuters shifting to alternative routes when higher-income users take advantage of the lower travel times upstream. The results imply that the non-identical assumption reflects route choice behaviour better. In addition, it could be expected that a different distribution would result if there were elastic demand.

Figure 3 Difference in traffic flows under first-best pricing
Non-identical users (NIU) – Identical users (IU)

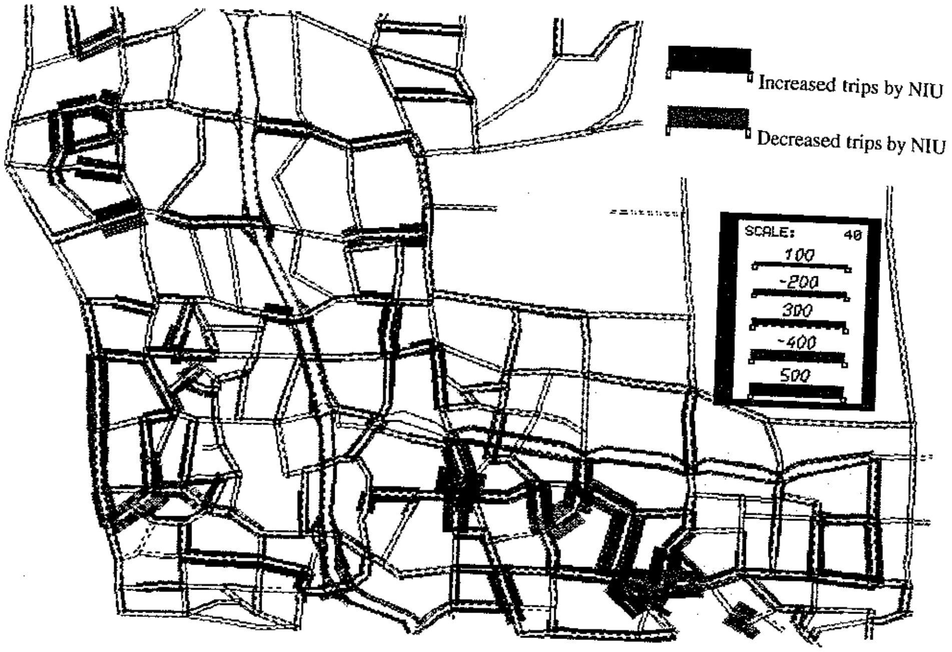
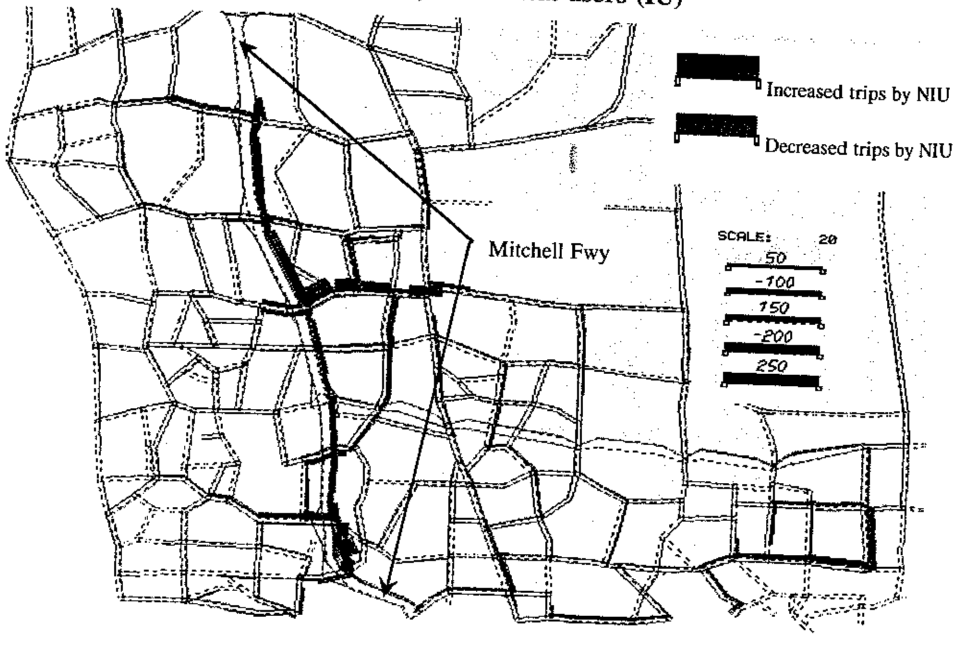


Figure 4 Difference in traffic flows under second-best pricing
Non-identical users (NIU) – Identical users (IU)



Conclusions

This paper reports on a detailed analysis of commuter route choice behaviour in the presence of optimal prices in both the simple network model and the general network case The pricing objective was to maximise society's welfare Three pricing regimes were examined: no pricing, first-best pricing, and second-best pricing The focus of the pricing study was to examine whether differential responses to tolls can be modelled by relating people's willingness to pay a toll charge relative to their income. Twelve income classes were considered in the multiple income class assignment, and the efficient tolls at the socially optimal flows were derived

The simulations reveal significantly different traffic diversions, and consequently different optimal toll patterns in the case of non-identical road users. In terms of welfare implications, the results indicate that the lower-income groups are the most affected by pricing. When all routes are subject to marginal cost pricing, low-income drivers are more worst off. On the other hand, the higher income groups are the likely winners, particularly when pricing is only applied on portion of the urban road network.

In summary, non-identical treatment of users based on income appears to improve the prediction of route choice diversions, particularly when only portion of the road network can be charged. Allowing differential responses to pricing has the potential to:

- ⇨ Provide better estimates of the optimal tolls, particularly the second-best optimum toll.
- ⇨ Provide better estimates of the winners and losers from road pricing (important in any consideration of political acceptance of road pricing).
- ⇨ A multiple income class approach seems to partly address some geographic differences in income distribution. This is advantageous in urban areas where more than one tolled facility exists.

The simplified multiclass user approach can be applied with other market segmentations of O-D demand, e.g. using values of time by trip purpose or by different vehicle classes.

Acknowledgment

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Appendix

Table A Route flows, optimal fees and time saved under three different pricing regimes

	Arterial Demand(1) (veh/hr)	Vol./Cap. Ratio	Motorway Demand(1) (veh/hr)	Vol./Cap. Ratio	First-best Fee (cents/km)		Second-best Fee(2) (cents/km)	% of First-Best Toll(3)	Time Saved (min)
					Arterial	Motorway	Motorway		
No pricing	174	12%	2826	94%	0	0	0	0	0
Demand 50/30/20									
Identical users	770	51%	2230	74%	4.1	9.2	4.7	50.6%	4.50
Non-identical users	828	55%	2172	72%	5.0	8.5	2.9	33.5%	5.08
Demand 35/35/30									
Identical users	770	51%	2230	74%	4.1	9.2	4.7	50.6%	4.50
Non-identical users	800	53%	2200	73%	4.6	8.9	3.8	42.3%	4.80
Demand 20/40/40									
Identical users	770	51%	2230	74%	4.1	9.2	4.7	50.6%	4.50
Non-identical users	777	52%	2223	74%	4.2	9.2	4.5	48.8%	4.57
Demand 10/20/70									
Identical users	770	51%	2230	74%	4.1	9.2	4.7	50.6%	4.50
Non-identical users	748	50%	2252	75%	3.7	9.5	5.3	56.0%	4.29

(1) Demand for travel on each route is equal under first-best and second-best pricing.

(2) The second-best fee is only charged on the motorway, the arterial route is left unpriced.

(3) Percentage of second-best fee relative to first-best fee.

Table B Comparison of total social and private costs

	Total social costs[1]		Total user-borne costs[2]			Loss to society			
	No pricing	Optimal	No pricing	First-best	Second-best	No Pricing	Percent	Second-best	Percent
Identical Users[3]	$22,350	$16,870	$13,844	$16,870	$14,679	$8,506	38.1%	$2,191	13.0%
Demand 50/30/20	$21,881	$15,976	$13,407	$15,976	$13,251	$8,474	38.7%	$2,725	17.1%
Demand 35/35/30	$24,539	$17,201	$14,742	$17,201	$14,743	$9,797	39.9%	$2,458	14.3%
Demand 20/40/40	$27,521	$18,607	$16,239	$18,607	$16,355	$11,282	40.0%	$2,252	12.1%
Demand 10/20/70	$31,061	$20,358	$18,037	$20,358	$18,349	$13,024	41.9%	$2,009	9.9%

(1) Sum of travel time + congestion costs + vehicle operating costs, i.e. the costs of travel to society.

(2) Sum of travel time + vehicle operating costs + toll charges.

(3) The same costs obtained for all demand mix.

Note that the vehicle operating cost is calculated at 10 cents/km.

Table C Total private variable costs by income group[1]

	Identical Users					Non-identical Users				
	No Pricing	FB Pricing	SB Pricing	FB-NP	SB-NP	No Pricing	FB Pricing	SB Pricing	FB-NP	SB-NP
Demand 50/30/20										
Low	$6,922	$8,435	$7,339	21.9%	6.0%	$4,870	$6,508	$5,146	33.6%	5.7%
Medium	$4,153	$5,061	$4,404	21.9%	6.0%	$4,447	$5,128	$4,311	15.3%	-3.1%
High	$2,769	$3,374	$2,936	21.9%	6.0%	$4,091	$4,339	$3,795	6.1%	-7.2%
Total	**$13,844**	**$16,870**	**$14,679**	**21.9%**	**6.0%**	**$13,407**	**$15,976**	**$13,251**	**19.2%**	**-1.2%**
Demand 35/35/30										
Low	$4,845	$5,905	$5,138	21.9%	6.0%	$3,418	$4,556	$3,696	33.3%	8.1%
Medium	$4,845	$5,905	$5,138	21.9%	6.0%	$5,244	$6,106	$5,246	16.4%	0.0%
High	$4,153	$5,061	$4,404	21.9%	6.0%	$6,080	$6,539	$5,802	7.6%	-4.6%
Total	**$13,844**	**$16,870**	**$14,679**	**21.9%**	**6.0%**	**$14,742**	**$17,201**	**$14,743**	**16.7%**	**0.0%**
Demand 20/40/40										
Low	$2,769	$3,374	$2,936	21.9%	6.0%	$1,931	$2,544	$2,093	31.7%	8.4%
Medium	$5,538	$6,748	$5,872	21.9%	6.0%	$5,974	$7,072	$6,172	18.4%	3.3%
High	$5,538	$6,748	$5,872	21.9%	6.0%	$8,333	$8,992	$8,091	7.9%	-2.9%
Total	**$13,844**	**$16,870**	**$14,679**	**21.9%**	**6.0%**	**$16,239**	**$18,608**	**$16,356**	**14.6%**	**0.7%**
Demand 10/20/70										
Low	$1,384	$1,687	$1,468	21.9%	6.0%	$991	$1,252	$1,051	26.3%	6.1%
Medium	$2,769	$3,374	$2,936	21.9%	6.0%	$2,987	$3,614	$3,212	21.0%	7.5%
High	$9,691	$11,809	$10,275	21.9%	6.0%	$14,058	$15,491	$14,085	10.2%	0.2%
Total	**$13,844**	**$16,870**	**$14,679**	**21.9%**	**6.0%**	**$18,037**	**$20,357**	**$18,348**	**12.9%**	**1.7%**

(1) Sum of travel time costs + vehicle operating costs + toll charges. Note that the value of time for identical users is $10.00 per hour. Values of time for non-identical users are those listed in Table 2. The vehicle operating cost is calculated at 10 cents/km.

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