Making the most of Assignment Models or is Iteration Boring?

Robert J Nairn BE, BEc, CPEng, FIEAust, LFITE

1. Introduction

This paper discusses how to get the best out of strategic traffic assignment models. A recent article showed that, of 210 projects studied, 50% of traffic forecasts were in error by 20% of their actual traffic flows[1]. The reasons ascribed to these errors were primarily trip generation and distribution or land-use development issues. However, while there are many sources of inaccuracy in making traffic forecasts with strategic models, this paper shows that inadequate use of the assignment sub-models contributes as well.

It is expected that an assignment model will return the same result if repeated (unless it is a "Stochastic" form of model with random choice cost variables) but this paper will show that it may be quite different if repeated with one more cycle, some traffic volumes being more than 25% different.

The paper is not about model calibration or validation, which is well catered for in the VicRoads guidelines[2] and other references. Nor is it about the accuracy of traffic forecasting with hindsight like the paper referred to above. It is about making sure that the traffic assignment process itself does not introduce unnecessary inaccuracies.

It focuses on the need for two forms of modelling iteration. They are

(a) Iteration within the assignment process, which is called "cycles" in this paper, and

(b) Iteration between trip generation and traffic assignment.

The main questions are "how many iterations or cycles are necessary?" to get the best results and "how do we tell when the best results are achieved?" A secondary question posed in the paper is "which is the best form of assignment model?"

These questions are addressed using a congested model of future Sydney, which possibly exaggerates the conclusions but clearly illustrates the principles involved.

2. Iteration within the assignment model

2.1. Introduction

Successive cycles within the assignment model are designed to achieve the Wardropp Principle, which states that equilibrium is achieved when, after taking congestion into account, the final trips face paths which are of equal cost.

A corollary is that, as all trips achieve their minimum travel cost, then the total travel cost within the network is a minimum and this is usually used as a guide to the number of cycles necessary for the assignment model.

Several different means of achieving this least-cost result are built into different types of assignment model. This paper discusses two types of assignment model as follows

---

[1] Flyvbjerg B, Holms M K S and Buhl S L (2006)

[2] VicRoads (2005)

(a) “Equilibrium” assignment (The “Volume Averaging” algorithm in TRIPS is just a special case of this type of model) in which the whole assignment is adjusted each cycle according to the congestion-affected paths traced after the previous iteration[3], and

(b) “Incremental” assignment in which the assignment is accumulated in steps, each step being assigned to paths traced from the previous cycle, which has been factored upwards to provide congestion effects.

The “Equilibrium” assignment method is in more common use (for example in Melbourne and Sydney) but “Incremental” assignment is also used quite frequently. Other forms of assignment are used (“Stochastic” assignments are not included in this paper because they are expected to give different traffic forecasts even for the same number of cycles or iterations) but this paper attempts to answer certain questions in the context of a comparison between just these two forms of assignment as follows:-

(a) How quickly do they converge to an acceptable limit for the total network travel cost?

(b) Are they internally consistent? That is how much do different link volumes vary between successive cycles when the model is considered to be adequately converged?

(c) Do the two methods need the same treatment?

2.2. Model convergence

The normal criterion for determining when the number of cycles used in an assignment model is adequate is that the eventual total network cost is close to its minimum value. The highway cost definition (“generalized cost”) should include the perceived value of travel time, vehicle operating costs, tolls and parking charges. In normal practise, a model may be stopped when the total network cost “gap” is, say, 1% or, alternatively, it may be stopped after a set number of cycles and the total network cost is no longer changing by a fixed limit.

To illustrate convergence, a model for Sydney was run in very congested afternoon peak conditions to establish the degree to which the total network costs (generalised cost) converged with successive cycles. The result is shown in Figure 1.

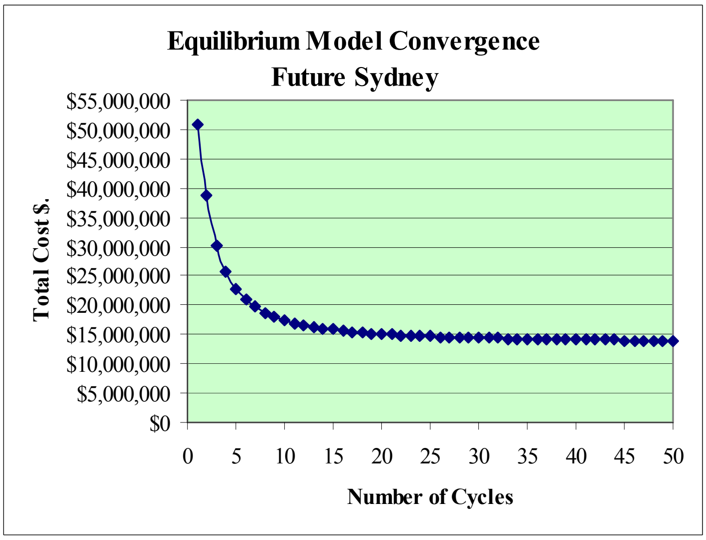

Figure 1 Assignment Model Convergence

---

[3] Luk, J.Y.K. and Wigan, M.R. (1977)

The model took almost 30 cycles to converge within 4% of its ultimate cost and over 40 cycles to converge within 1%. It is unusual for as many as 30 cycles to be used in common practice but it is very unlikely that a convergence within 4% would be considered as an acceptable result.

The Sydney network used to derive Figure 1 was expected to be heavily congested with about 25% of the roads in the network having Volume/Capacity (V/C) ratios greater than 0.9 in the afternoon peak hour. The above result is, therefore, due to the heavy level of congestion in this future Sydney model. The number of iterations to achieve a target closure depends on the level of congestion in the network.

To further illustrate this principle, figure 2 shows convergence of a Perth model, which has been artificially set with different levels of congestion, shown as the percentage of the length of the network with V/C ratios greater than 0.90.

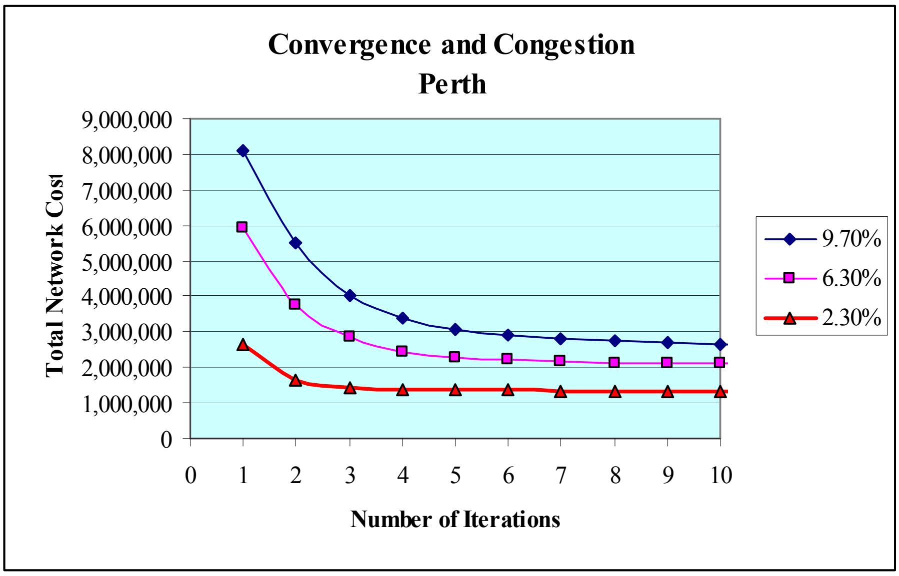

Figure 2 Convergence and Congestion Levels

This Perth model converged within 3% in 9 iterations when 9.7% of the length of the network was congested, 7 iterations for 6.3% congested and 5 iterations for 2.3% of the network congested. Clearly, the number of assignment cycles must increase as the level of congestion increases.

This illustrates that it is not sufficient to be satisfied with the number of iterations applied during calibration or in the “base year case” if the same number is to be used for forecasting future years when considerably more congestion may ensue.

2.3. Incremental and equilibrium model convergence

Figure 3 shows a comparison between the convergences of an incremental assignment model with an equilibrium model, both calibrated for Sydney on the identical network with the identical land-use and travel parameters. Calibration compared trip generation, trip length frequency and mode split as well as traffic volumes in both cases.

For exactly the same congested conditions in the Sydney network, the incremental model took 15 cycles to converge within 4% of its ultimate cost and 27 cycles to converge within 1% compared with 29 and 42 for the equilibrium model.

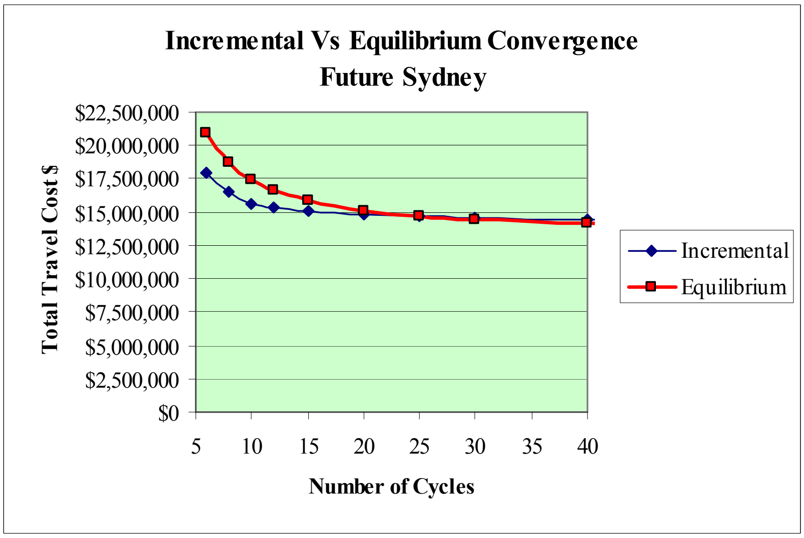

Figure 3 A comparison between the Convergence of Equilibrium and Incremental Models

2.4. Consistency

It may not sufficient to rely on model convergence characteristics as the sole criteria of a satisfactory assignment result. This is because the network total cost may have reached a level close to its minimum but some traffic volumes may still be switching between routes of approximately equal cost. Therefore another test is suggested – here called "consistency".

The consistency of an assignment refers to the extent to which link volumes change between successive cycles. If they change by a significant amount then there cannot be much confidence in the resulting forecasts and further cycles may be necessary even though there is adequate convergence of the cost criterion.

Table 1 shows the average percent difference in predicted traffic on all links in the network compared with one more cycle for different levels of traffic volume using the same congested Sydney model. Results for links with less than 1,000 vehicles per hour are not included. There were over 8,000 links and 839 zones in the Sydney network used for these tests.

Table 1 Average Difference between Successive Cycles for Incremental and Equilibrium Assignment Models for Different Volume Ranges

One-way Volume per hour	Between Cycle 9 and 10		Between Cycle 15 and 16		Between Cycle 24 and 25	
	Incremental	Equilibrium	Incremental	Equilibrium	Incremental	Equilibrium
> 10,000	0.79%	0.57%	0.26%	0.11%	0.20%	0.17%
9,000 to 9,999	0.82%	5.36%	0.79%	3.67%	0.31%	0.87%
8,000 to 8,999	1.46%	4.78%	1.23%	2.02%	0.37%	0.50%
7,000 to 7,999	1.54%	3.32%	0.90%	1.76%	0.26%	0.42%
6,000 to 6,999	1.33%	2.13%	0.65%	2.35%	0.24%	0.49%
5,000 to 5,999	2.14%	2.97%	0.77%	2.15%	0.45%	0.77%
4,000 to 4,999	2.05%	3.38%	0.94%	2.73%	0.45%	0.97%
3,000 to 3,999	2.64%	3.64%	0.96%	3.06%	0.56%	1.28%
2,000 to 2,999	2.60%	4.12%	1.09%	3.42%	0.61%	1.43%
1,000 to 1,999	3.64%	3.49%	1.65%	3.30%	1.10%	1.98%
Overall Average	3.62%	3.47%	1.62%	3.16%	0.99%	1.74%

Table 1 shows that, as the cycle number increases, there is less difference between successive cycles with the incremental assignment model than with the equilibrium model. Further, it shows that, in general, the incremental model changed less for high volumes than the equilibrium models.

Table 2 shows this comparison in a different way, when link volumes are compared after three sets of successive cycles for the congested Sydney model. The proportion of links having the stated percent difference between the link volumes after the 9th cycle compared with the 10th cycle, the 12th cycle and the 13th and the 15th and 16th is listed.

Table 2 Percent of Links between Successive Cycles for Incremental and Equilibrium Assignment Models with Different Ranges of Change

	9 to 10 Cycles		12 to 13 Cycles		15 to 16 Cycles	
Range	Incremental	Equilibrium	Incremental	Equilibrium	Incremental	Equilibrium
> 25%	1.60%	0.81%	0.70%	1.43%	0.39%	0.48%
20% to 25%	0.95%	0.91%	0.15%	0.81%	0.30%	0.37%
15% to 20%	1.79%	1.93%	0.51%	2.27%	0.40%	1.47%
10% to 15%	3.60%	5.42%	1.24%	6.49%	0.92%	4.49%
5% to 10%	12.27%	17.46%	6.08%	19.31%	3.31%	13.24%
1% to 5%	46.81%	46.09%	43.23%	50.60%	34.60%	37.90%
< 1%	32.99%	27.38%	48.10%	19.09%	60.06%	42.06%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Table 2 shows that, even though the Incremental Sydney model had converged to within approximately 4% after the 15th cycle, 8.7% of the link volumes change by more than 5% when this assignment is run for an additional 13th cycle. By contrast, 30% change by more than 5% in the 13th cycle with the equilibrium assignment.

The test for consistency, particularly for links that may be under investigation, is therefore quite useful and may be essential, particularly when using equilibrium assignment models. Table 3 shows the proportion of links in the network whose traffic volumes vary by 10% or more between successive cycles.

Table 3 Percent of Volumes different by more than 10% of Previous Cycle

No of Cycles	Incremental	Equilibrium
9 to 10	7.9%	9.1%
12 to 13	2.6%	11.0%
15 to 16	2.0%	6.8%
24 to 25	1.1%	2.6%

This is because the incremental model copes better with congested conditions, which is illustrated in table 4, which shows the difference in volumes between the successive cycles of the Sydney model classified by congestion level.

Table 4 Average Difference between Successive Cycles for Incremental and Equilibrium Assignment Models for Different Volume to Capacity Ranges

Volume/Capacity Ratio	12 to 13 Cycles		15 to 16 Cycles	
	Incremental	Equilibrium	Incremental	Equilibrium
> 1.0	1.96%	2.35%	0.75%	1.66%
0.95 to 0.99	2.35%	2.69%	0.91%	1.99%
0.90 to 0.94	2.72%	3.02%	1.10%	2.13%
0.85 to 0.89	3.33%	2.73%	1.48%	2.49%
0.80 to 0.84	3.69%	3.25%	1.62%	2.13%
0.70 to 0.79	4.76%	3.42%	1.83%	2.71%
0.60 to 0.69	5.89%	4.13%	2.54%	3.30%
0.50 to 0.59	7.53%	4.07%	2.94%	3.30%
0.25 to 0.49	9.21%	3.83%	3.23%	3.30%
< 0.25	5.25%	2.65%	4.94%	1.90%

The Incremental model changes less between cycles for high V/C ratios, which are most likely to be the links under investigation, even though, for fewer cycles, it has greater changes for less congested roads.

It is for this reason that the incremental assignment method is preferred by some practitioners over the equilibrium method. Road links under investigation may still be changing volume by a significant margin after each equilibrium cycle

3. Iteration between trip generation and assignment

3.1. Introduction

Travel is an economic function so travel demand is responsive to travel costs. That is, it is just as logical to expect congestion to suppress travel as it is to expect generated travel from new transport infrastructure. It means that trip generation is partly an output of a strategic transport model not just a fixed input. Similarly congestion may affect trip distribution or mode choice. Trip matrices need to be balanced against the influences of congestion in the networks.

The method of representing this concept in modelling form is illustrated in figure 4 in which congested skim files, created by the assignment model, are fed back into the trip generation, trip distribution and mode split models.

Therefore this part of the paper is not relevant to those who use “Matrix Estimation” or so-called “Iterative Validation” with subsequent matrix expansion methods. It has been pointed out that this method destroys any logic used in the process of producing the trip matrices[4] but it also loses control of the total network costs, even during the “calibration” process, so there is no indication that Wardropp’s principle has been achieved in the overall modelling process. It attempts to adjust the trip tables even for errors or inaccuracies in the network coding, rather than pointing to them during calibration, and it cannot reflect increasing congestion in the trip generation, distribution or mode split models.

---

[4] Nairn R J (2004)

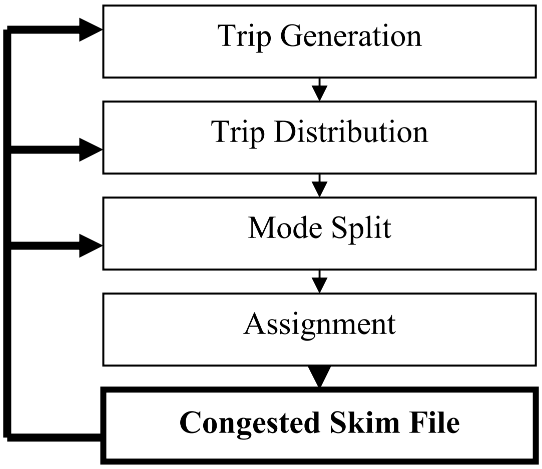

Figure 4 Iteration between Trip Generation and assignment

The need for this form of modelling iteration was first illustrated by the Australian Road Research Board[5] and, although neglected by many, it is now becoming accepted practice with some practitioners. This paper emphasises the need for this form of modelling.

3.2. Convergence - How much iteration is needed?

An extension of the Wardropp Principle is that this trip generation/congestion process should achieve the minimum total network travel cost. Therefore this criterion should also be used to decide when to stop this iteration process. Table 5 shows how the Sydney model converged for both the incremental and equilibrium types of assignment model. This test used 50 assignment cycles.

Table 5 Percent Difference in Total Network Costs after successive Iterations

No. of Iterations	Incremental	Equilibrium
12	6.33%	19.68%
14	0.51%	9.97%
16	-0.41%	5.07%
18	0.02%	2.79%
20	-0.50%	1.06%
22	-0.49%	0.37%
24	-0.03%	0.22%
26	-0.10%	0.02%
28	0.18%	0.00%
30	-0.38%	
40	-0.13%	
50	0.00%	

The incremental model converged to within 0.5% in 16 iterations whereas the equilibrium model took 22 iterations. However, the equilibrium model stabilized completely after 28 iterations whereas the incremental model was still showing minor differences after 40 iterations. Figure 5 illustrates the convergence for both models.

---

[5] Luk, J.Y.K., Nairn, R.J. and Parker, G.R. (1978)

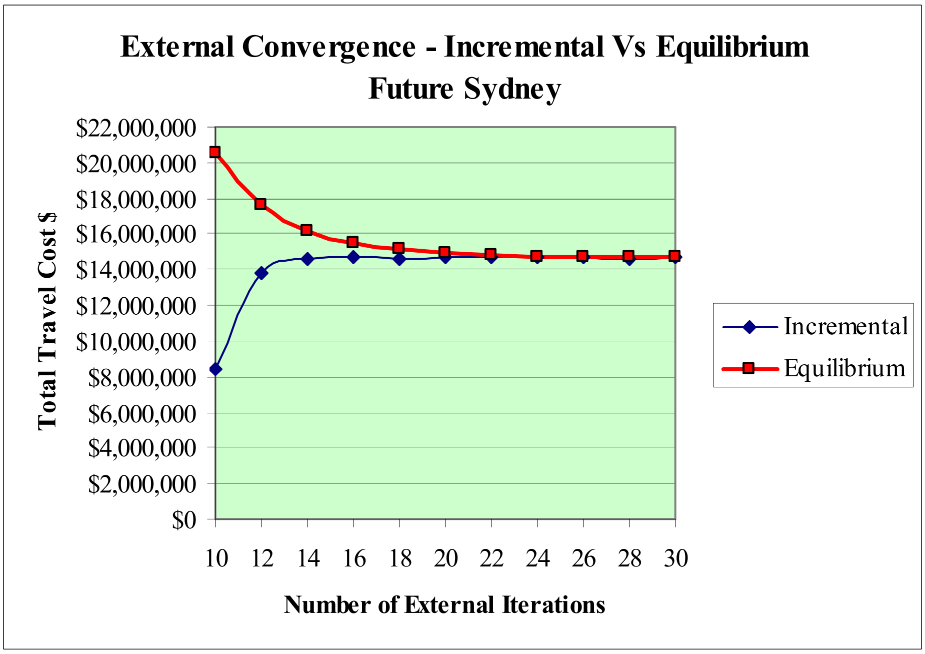

Figure 5 Trip Generation/Congestion Convergence of Incremental and Equilibrium models

It is necessary to use dampening to help the modelling process converge. This may involve averaging the successive skim or trip matrices or both. In deriving table 5 or figure 5 the same form of dampening was used for both the model using incremental assignment and that using equilibrium assignment. The same number of assignment cycles was used for each. The different way in which the models converged may be due to the dampening process.

3.3. Consistency

The consistency test may also be applied to check if this form of Trip Generation/Congestion convergence is satisfactory. Table 6 lists the results.

Table 6 Average Difference between Successive Iterations using Incremental and Equilibrium Assignment Models for Different Volume Ranges

One-way Volume	Between Iteration 12 and 13		Between Iteration 15 and 16		Between Iteration 24 and 25	
per hour	Incremental	Equilibrium	Incremental	Equilibrium	Incremental	Equilibrium
> 10000	0.19%	2.31%	0.17%	0.80%	0.39%	0.16%
9,000 to 9,999	2.29%	1.96%	0.66%	0.34%	0.41%	0.29%
8,000 to 8,999	2.00%	0.88%	0.37%	0.72%	0.92%	0.68%
7,000 to 7,999	1.95%	1.20%	0.45%	0.65%	0.39%	0.84%
6,000 to 6,999	1.83%	1.68%	0.74%	0.91%	0.41%	0.71%
5,000 to 5,999	2.46%	1.53%	0.69%	0.90%	0.63%	0.98%
4,000 to 4,999	1.84%	2.31%	0.74%	1.42%	0.61%	1.14%
3,000 to 3,999	2.43%	2.26%	0.90%	1.62%	0.78%	1.25%
2,000 to 2,999	2.72%	2.51%	1.06%	1.92%	0.78%	1.41%
1,000 to 1,999	5.12%	3.00%	1.97%	2.46%	1.46%	1.93%
Overall Average	4.44%	2.73%	1.78%	2.16%	1.28%	1.76%

Table 6 shows that significant differences can occur between successive iterations (using the same number of cycles) and that, while the incremental sub-model has greater average differences than the equilibrium sub-model for a smaller number of iterations, it improves with more iterations.

The figures in table 6 are averages for all links in the stated volume range but some links will vary much more than the average. Table 7 shows the proportion of links whose traffic volumes vary by 10% or more between successive iterations for both the model using incremental assignment and that using equilibrium assignment.

Table 7 Percent of Volumes with differences of more than 10% 0f Previous Iteration

No of Iterations	Incremental	Equilibrium
12 to 13	10.8%	11.0%
15 to 16	2.5%	8.2%
24 to 25	1.3%	2.9%

This consistency check showed that the model using equilibrium assignment methods would need more iteration to provide the same consistency as the incremental model.

4. Combined effect of iterations and assignment cycles

Figures 6 and 7 show the effect on convergence with increasing assignment cycles for both models.

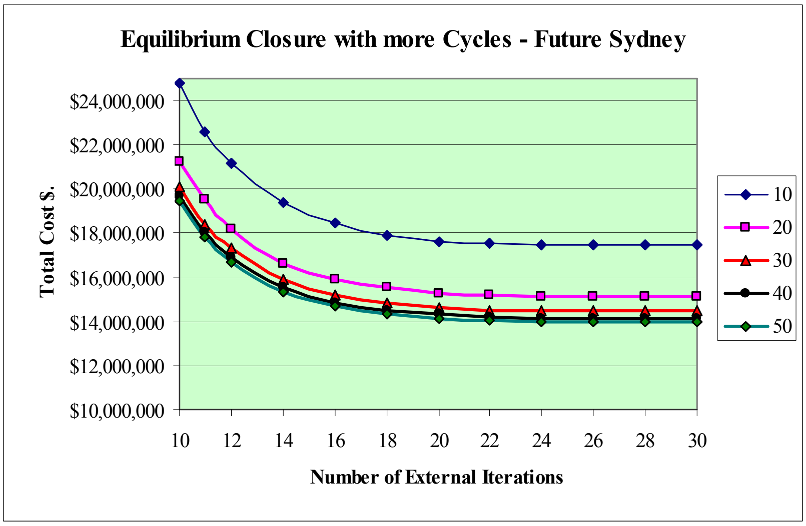

Figure 6 Convergence with increasing Equilibrium Assignment Cycles

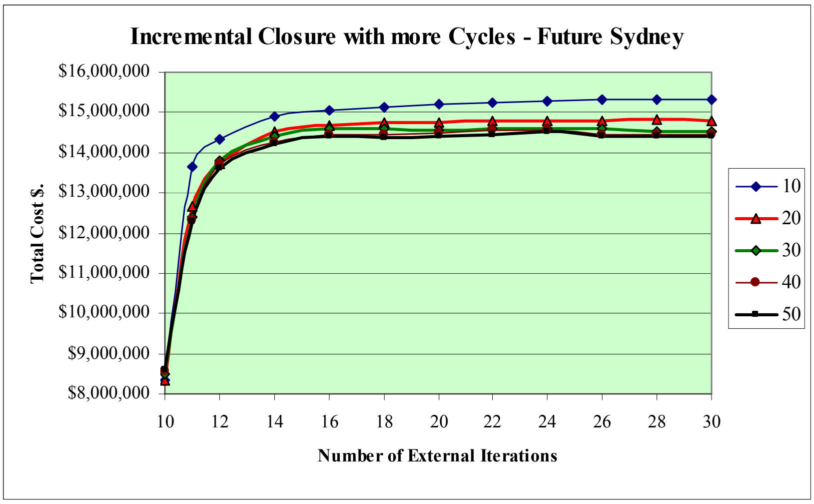

Figure 7 Convergence with increasing Incremental Assignment Cycles

Both figures 6 and 7 show that, even with as many as 50 cycles from either the equilibrium or incremental models, the total network cost still diminishes slightly with successive external iterations. For the Sydney model the adverse effect of using inadequate assignment cycles appears to be more severe for the model using equilibrium assignment.

This also shows that, if total network cost is an important output of the assignment process, for economic evaluation or other reasons, then a substantial number of both cycles and iterations are necessary to achieve reliable results.

5. What happens to the trip table?

The iteration process changes trip generation, trip distribution and mode split as well as the assignment and, although this paper is primarily about accuracies in the assignment process, it is helpful to understand what is happening to the trip table used in the assignment as the iteration progresses.

To examine what might be happening to trip generation and trip distribution during the iteration closure for both types of assignment, the zonal trip attractions were compared after the 10th, 15th, 20th and 30th iteration so see how much individual zone attractions varied.

Table 8 Percent changes in trip attractions in the trip tables between the stated number of iterations

% difference in Zonal Attr after stated Iterations	Incremental Assignment			Equilibrium Assignment		
	10 to 15	15 to 20	20 to 30	10 to 15	15 to 20	20 to 30
10% to 15%	32.73%	0.00%	0.00%	0.00%	0.00%	0.00%
5% to 10%	28.15%	0.98%	0.00%	0.64%	0.00%	0.00%
2% to 5%	20.79%	16.04%	0.00%	50.08%	0.00%	0.00%
1% to 2%	9.98%	27.82%	1.64%	33.70%	5.09%	0.00%
< 1%	8.35%	55.16%	98.36%	15.58%	94.91%	100.00%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Note: Only zones with 500 or more trip attractions were included in this analysis

Table 8 shows that, for the Sydney model, the trip distribution closes more rapidly with the equilibrium assignment sub-model than with the incremental sub-model. The corollary is that the link volume changes that occur between successive iterations are more the result of the assignment sub-model than trip table changes, when using for the equilibrium sub-model. When the incremental sub-model is used, trip table changes contribute more to the lack of assignment consistency.

6. Conclusions

The following conclusions are drawn from this work:-

(a) There can be very substantial differences between traffic volumes in successive assignment cycles or trip generation/congestion iterations;

(b) The necessary number of assignment cycles increases substantially with congestion and model calibration gives no guidance to how many are required in future forecasts;

(c) Both equilibrium and incremental models need more cycles for the total network travel cost to converge within acceptable convergence (say 1%) than is usually applied in practice;

(d) Both equilibrium and incremental models need a substantial number of trip generation/congestion iterations to achieve an acceptable degree of network cost convergence;

(e) A consistency check, as defined in this paper, is a necessary safeguard against assignment error and may result in applying more cycles or iterations than is indicated by travel cost convergence; and

(f) If total network cost is an important output of the assignment process, for economic evaluation or other reasons, then a substantial number of both cycles and iterations are necessary to achieve reliable results.

There are differences between the equilibrium and incremental models as follows:-

(a) The incremental model will take fewer cycles for the appropriate convergence of total network cost;

(b) The incremental assignment model should be more consistent for high volumes and high V/C ratios than the equilibrium assignment model; and

(c) Trip distribution closes more rapidly with the equilibrium assignment sub-model than with the incremental model. The corollary is that the link volume changes that occur between successive iterations are more the result of the assignment sub-model than trip table changes, when using the equilibrium sub-model. When the incremental sub-model is used, trip table changes contribute more to the lack of assignment consistency.

The paper does not attempt to provide detailed guidelines about the number of iterations or cycles that are necessary to ensure adequate convergence and consistency because the answer depends on the level of congestion. In addition, as this research into this type of assignment behaviour has been carried out using only a very congested Sydney model (2011 or thereabouts) more research is needed to confirm the findings in different assignment environs.

However, it shows that it is necessary to carry out tests on convergence and consistency to avoid the risk of large potential errors caused by the use of the assignment sub-model alone.

7. References

Flyvbjerg B, Holms M K S and Buhl S L (2006) *Inaccuracy in Traffic Forecasts*, Traffic Reviews Vol 26 No 1 1-24 January 2006

VicRoads (2005) *Draft Guidelines on Model Validation Process and Criteria* January 2005

Nairn R J (2004) *Modelling or Muddling* 27th Australasian Transport Research Forum, Adelaide, 29 September - 1 October 2004

Luk, J.Y.K. and Wigan, M.R. (1977) *ARRBTRAFIC Program User Manual - Equilibrium Assignment for Fixed Demands*. Australian Road Research Board Technical Manual ATM No. 4.

Luk, J.Y.K., Nairn, R.J. and Parker, G.R. (1978) *Interaction between Land-use/Distribution and Assignment*. Australian Road Research Board AIR 1059-1.