Introducing convergent feedback in the Melbourne Integrated Transport Model

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1 Introduction

The importance of achieving convergence in four-step transport models relates to the need to provide stable, consistent and robust model results. When model outputs are being used to compare the ‘base’ and ‘option’ scenario for a given infrastructure project (and subsequently to estimate the economic cost-benefit of the project), it is important to be able to distinguish real differences from those associated with different degrees of model convergence and spurious mathematical effects. Similar considerations apply when the benefits of different projects are being compared. Model convergence is therefore a key to robust transport planning and economic appraisal.

The term ‘model convergence’ may relate either to the assignment step in four-step models, or include feedback between the assignment step and trip distribution (or even trip generation). In the context of this work, convergence relates to feedback between the assignment step and trip distribution.

The practice amongst some transport modellers is to use a naïve feedback (Boyce et. al. 2004) procedure to solve the four step transport model. Such a procedure may be summarised as follows:

1. Generation of an initial zone to zone travel time matrix by ‘skimming’ the highway network using minimal cost routes and assuming zero link flows;
2. Establishment of an origin-destination (OD) matrix using the zonal trip-generation data and skimmed costs;
3. Assignment of the origin-destination matrix to the road network, and re-calculation of minimal cost routes;
4. Use of the updated cost matrix to re-establish the OD matrix followed by a re-assignment to the road network; and
5. Repeat of step 4 for a pre-determined fixed number of feedback cycles.

The above procedure does not invoke any tests to determine whether the final link volumes and travel times are consistent with the assigned OD matrix. By comparison, a convergent feedback procedure differs from naïve feedback in several important respects that include an averaging step and a convergence check (which provides a well-defined stopping criterion). The essential feature of convergent feedback in the four step process is that the input times used in the distribution step equal or approach the output times from the assignment step with each successive cycle. Boyce and Bar-Gera (2005) provide an overview of various solution procedures.

The model used in this work is a large and operational integrated transport model referred to as the Melbourne Integrated Transport Model (MITM). MITM is a traditional four step model containing over 2,200 zones and approx. 33,500 road links (Cube/Trips software platform). Notable features of MITM include the following:

- The generation step produces average-working-weekday person trip ends for each of 14 trip purposes. Freight travel is not included;
- The distribution step produces trip matrices for each purpose and applies trip departure time factors to produce a single 7-9am trip matrix;
- The mode choice step divides the 7-9am matrix into walk trips, public transport trips (on-road and off-road modes) and private vehicle person trips. The latter matrix is converted to vehicle trips using occupancy factors; and
- The assignment step assigns the public transport and private vehicle trip matrices to the respective networks.

The post assignment cost skims for each mode are then used in the mode choice step and a blended cost skim matrix for the distribution step in the subsequent cycle.

Whilst the model considers both private road travel and public transport trips, only the private road travel component is documented here.

This work seeks to:

1. Assess the effect of using naïve feedback in MITM;
2. Implement a convergent feedback procedure for MITM; and
3. Assess the benefits of using a convergent feedback procedure for MITM.

2 Results using naïve feedback

The total number of highway vehicle-driver trips (i.e. passengers not included), total vehicle kilometres travelled (VKT) and total vehicle minutes travelled (VMinT) as calculated using a naïve feedback procedure in MITM are presented in Table 1 for year 2001 and in Table 2 for 2031. The 2001 model was run for ten cycles, but because of time constraints, the 2031 model run was stopped after six cycles. In this work, the term ‘cycle’ refers to the feedback process between the assignment step and trip distribution.

The cycle number should not be confused with the number of iterations specified within the assignment stage of the four step process. For assignment, a fixed number of iterations using the equilibrium method (100 in total) were conducted each time to ensure adequate convergence within this step.

Previous modelling by VicRoads (via MITM) has demonstrated that in cases where the ‘no-build’ and ‘build’ scenarios are terminated at odd and even assignment-iteration number respectively, substantive travel time benefits may be calculated many tens of kilometres away from the project, even in cases of relatively small road project changes. Such results are not consistent with engineering judgement and are indicative of poor convergence of the assignment step. Stopping the assignment iteration process at the same number of iterations has been found to reduce the spatial extent of such apparent benefits.

Although the assignment step is sufficiently well converged for the present work (see section 4), these differences can in principle be minimised by improving assignment convergence. The slowness of the equilibrium method to converge precludes improving convergence using this method in practice, but see Slavin *et. al.* (2006) for a discussion of alternative algorithms and Boyce *et. al.* (2004) for the use of origin-based algorithms.

Table 1 Naïve feedback. MITM model results, 2001 (7-9am period)

	Interzonal Trips only		
Cycle	Total Veh. Trips (million)	Total VKT (million)	Total VMinT (million)
1	1.15	15.1	27.2
2	1.13	13.3	19.3
3	1.16	15.3	29.4
4	1.12	12.9	18.1
5	1.17	15.7	32.5
6	1.11	12.3	17.0
7	1.17	16.1	34.9
8	1.10	12.0	16.3
9	1.18	16.3	36.3
10	1.10	11.8	15.9

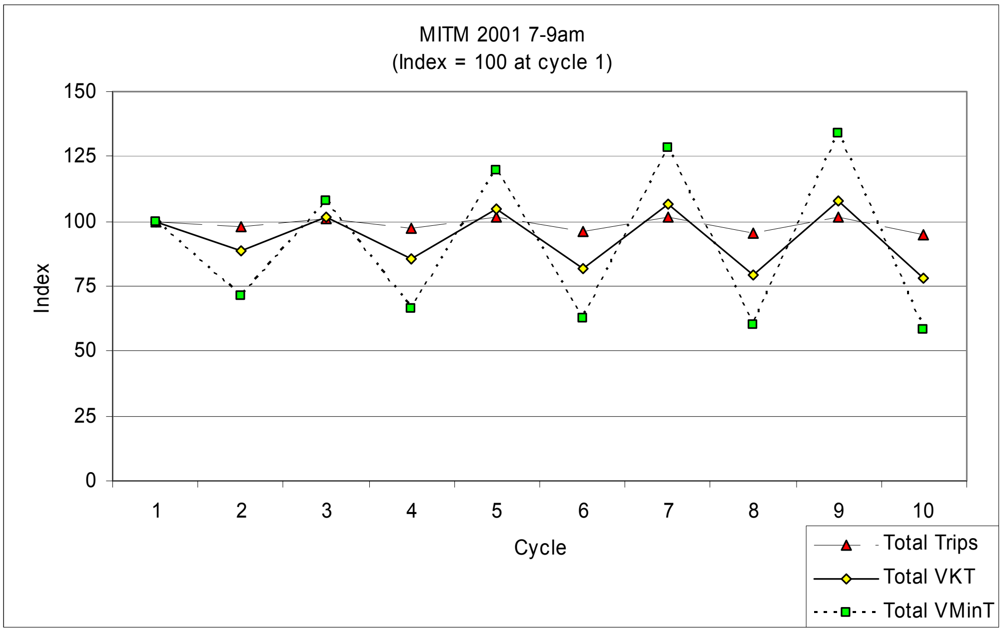


Figure 1 Naïve feedback. MITM 2001 (7-9am). Results indexed to the value of the first cycle

Table 2 Naïve feedback. MITM model results, 2031 (7-9am period)

	Interzonal Trips only		
Cycle	Total Veh. Trips (million)	Total VKT (million)	Total VMinT (million)
1	1.20	14.7	19.6
2	1.54	22.3	50.5
3	1.41	15.5	19.8
4	1.55	23.6	74.4
5	1.35	13.8	17.4
6	1.56	24.0	80.1

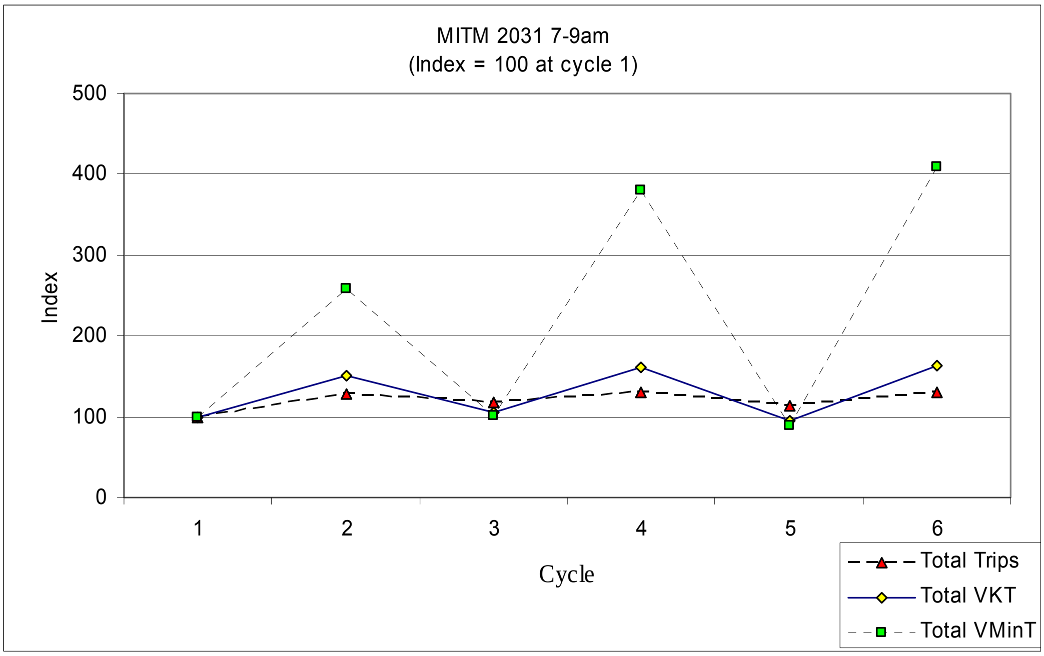


Figure 2 Naïve feedback. MITM 2031 (7-9am). Results indexed to the value of the first cycle

The model results presented in Figures 1 and 2 clearly diverge in an oscillatory manner from cycle to cycle, with divergence being greater in 2031 than in 2001. The degree of oscillation is least pronounced for the total number of vehicle trips and most pronounced for VMinT.

Given that the sum of private vehicle and public transport trips is constant; the oscillation in private vehicle trips indicates that the mode choice step is allocating a greater or lesser proportion of trips to public transport in each cycle.

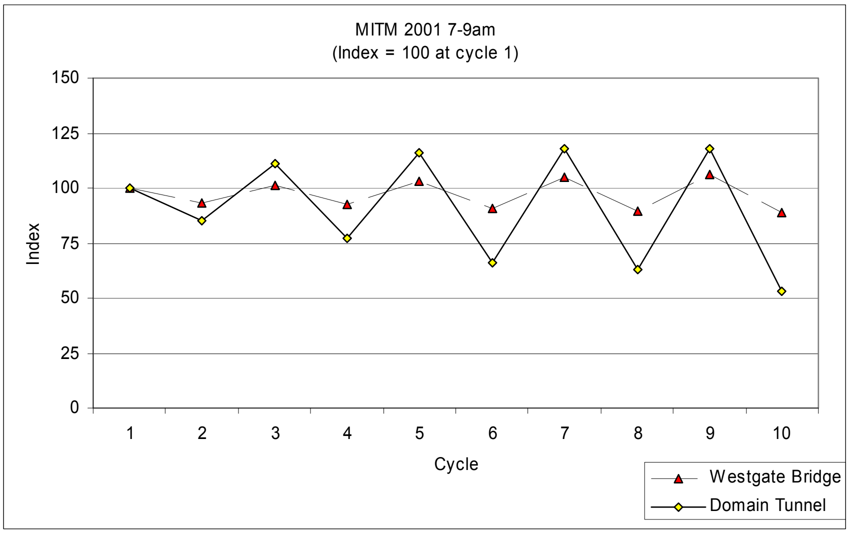
In Table 3, results are presented for the assigned link volumes by cycle number for two major road sections in Melbourne, namely the Westgate Bridge and the CityLink Domain Tunnel. Once again, substantive oscillatory fluctuations result from one cycle to the next.

The use of naïve feedback in MITM is found to result in divergent oscillatory behaviour for various model results. The number of highway trips, assigned link volumes and link travel times vary between cycles in an increasing manner and the differences after only a few cycles become very large. The modelled city-bound volume in the Domain Tunnel in 2001 for example, varies between 5 and 7 thousand vehicles (between 7 and 9am) between the fourth and fifth cycle. Similarly, the forecast city-bound volume over the Westgate Bridge in 2031 varies between 25 and 5 thousand vehicles between the fourth and fifth cycle.

Clearly, the use of naïve feedback can lead to very different economic and road planning conclusions depending on how many cycles the model is run for. For this reason, naïve feedback should not be used in large regional models used for road project assessments, particularly in cases of relatively high traffic congestion.

Table 3 Naïve feedback. Assigned link volumes (7-9am; thousands of vehicles) by cycle for two major road sections in Melbourne in 2001 and 2031

	Westgate Bridge (City-bound)		CityLink Domain Tunnel (City-bound)	
Cycle	2001	2031	2001	2031
1	17.6	17.9	6.2	5.1
2	16.4	20.7	5.3	8.8
3	17.8	12.6	6.9	3.3
4	16.3	25.0	4.8	9.7
5	18.1	5.2	7.2	1.6
6	16.0	25.4	4.1	9.8
7	18.5	-	7.3	-
8	15.8	-	3.9	-
9	18.7	-	7.3	-
10	15.6	-	3.3	-



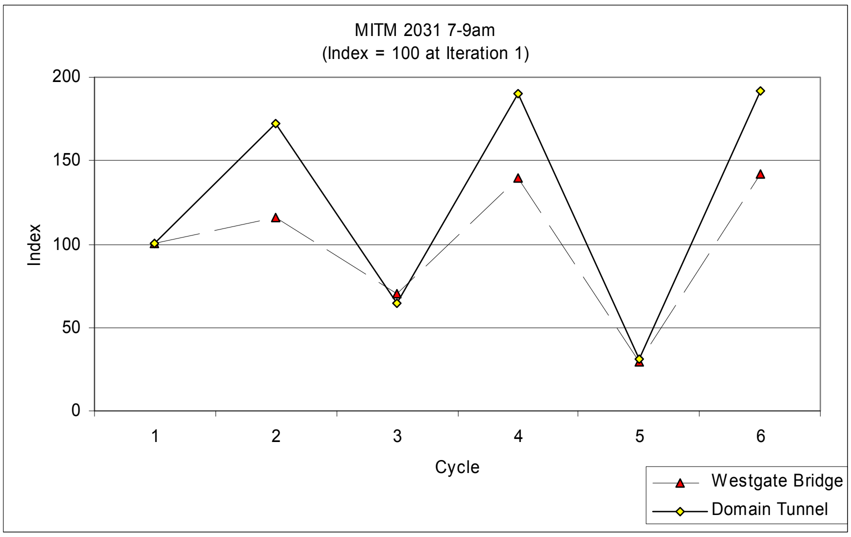


Figure 3 Naïve feedback MITM results for Westgate Bridge and CityLink Domain Tunnel in 2001 (top) and 2031 (bottom).

3 Modifying MITM to produce a convergent feedback solution

An assessment of the effect of different types of feedback procedures as applied to a sketch planning transport model (approx. 300 zones and 3000 highway links) has been presented by Boyce *et. al.* (1994). Boyce *et. al.* reported that by modifying the feedback procedure to include the averaging of the highway link flows (using the Method of Successive Averages, MSA) before running a cost skim, and feeding the new updated costs back into the distribution step, convergence can be assured.

In this work, the MSA procedure and skimming steps as proposed by Boyce et al have been incorporated into the structure of MITM (shown in Figure 4[1]).

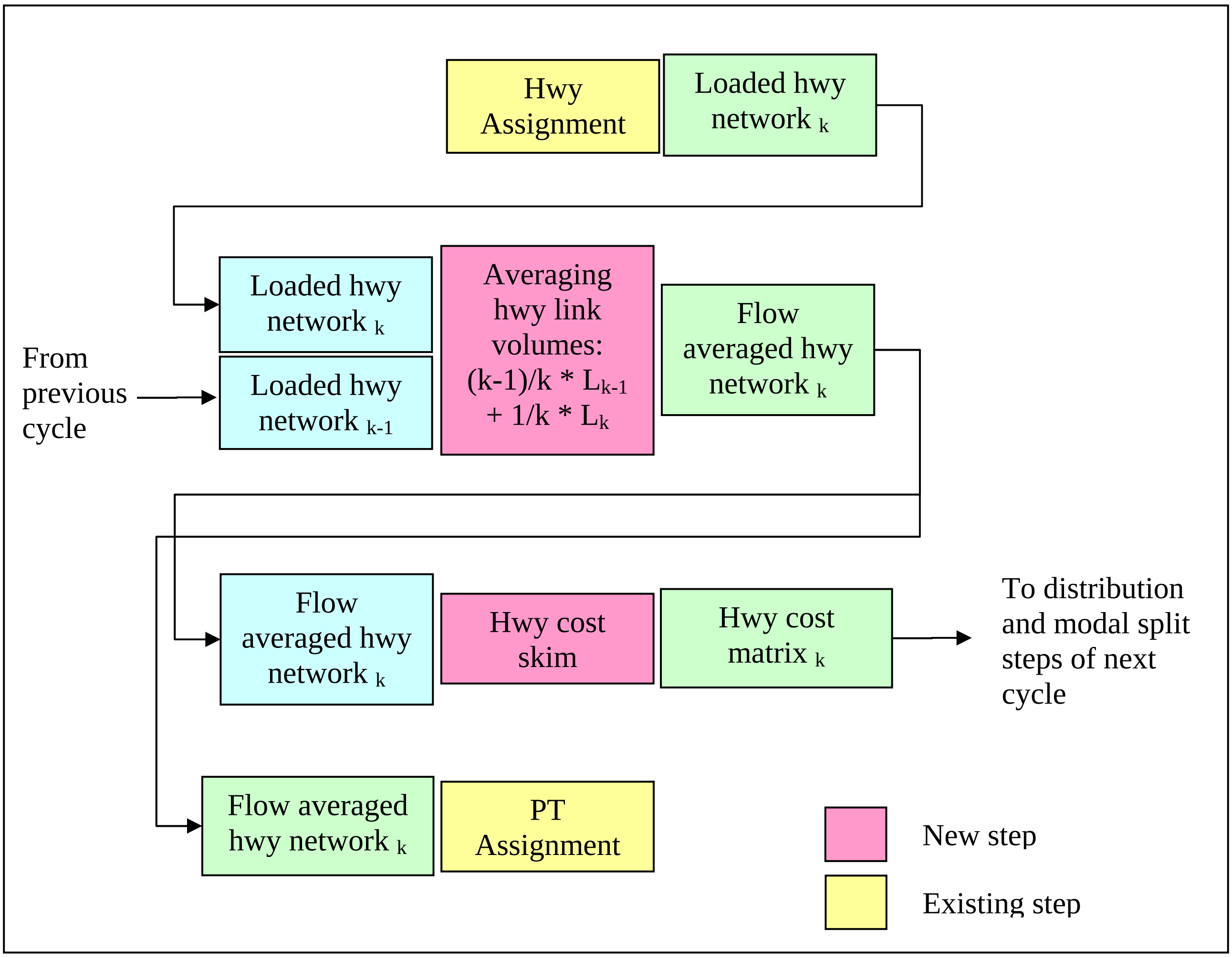


Figure 4 Block diagram of how the averaging and skimming steps were incorporated into the MITM structure

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[1] The new averaging and skimming steps were written as CAPRES blocks within the TRIPS MVHWAY hwy assignment program

4 Results using convergent feedback

After modification of the MITM feedback procedure, as presented in Figure 4, the model was run for a number of cycles. Results for the total number of highway trips, total vehicle kilometres travelled and total vehicle minutes travelled are presented in Table 4 for 2031.

Table 4 Convergent feedback. MITM model results, 2031 (7-9am period)

		Interzonal Trips only		
Cycle	Terminating δ (%)	Total Veh. Trips (million)	Total VKT (million)	Total VMinT (million)
1	0.1	1.20	14.7	19.6
2	1.6	1.37	22.3	50.5
3	0.5	1.42	20.1	33.1
4	0.6	1.44	20.0	32.6
5	0.5	1.45	19.8	31.9
6	0.6	1.46	19.7	31.5
7	0.5	1.47	19.6	31.3
8	0.5	1.47	19.6	31.1
9	0.5	1.48	19.6	31.0

Table 4 also presents the results for the Terminating *δ (*also known as the Relative Gap statistic*)*, which is often used to measure how well the assignment step has converged (see Ortuzar and Willumsen (2001) for more details).

$$\delta = \frac{\sum_{ijr} T_{ijr}\left(C_{ijr} - C_{ij}^{*}\right)}{\sum_{ij} T_{ij} C_{ij}^{*}} \qquad (1)$$

where:
T_{ij} is the number of trips from zone i to zone j,
T_{ijr} is the number of trips from zone i to zone j on path r,
C_{ij}^* is the minimum cost of travel from i to j, and
C_{ijr} is the cost of travel from zone i to zone j on path r.

C_{ijr} - C_{ij}^* is the excess cost of travel over a particular route relative to the minimum cost of travel for each (i, j) pair. Therefore *δ* is a measure of the total cost of excess travel via less than optimal routes, with the denominator introduced so that the measure is recorded in relative rather than absolute terms.

The terminating *δ* of less than 1% in Table 4, as calculated for each assignment step by the TRIPS software after 100 iterations, shows that costs are within 1% of the completely converged solution at each cycle. This degree of assignment convergence is generally considered good modelling practice; for instance meeting the UK Highways Agency (1996) criteria. Furthermore because the deviation in VKT and VMinT by cycle (which are the primary components of cost) are substantially greater than 1% when using naïve feedback, and less than 1% when implementing convergent feedback, it may be concluded that the assignment step is sufficiently well converged so as not to mask the benefits of implementing feedback in this work.

The interzonal data in Table 4 are plotted in Figure 5 with the results indexed to the value at the first cycle.

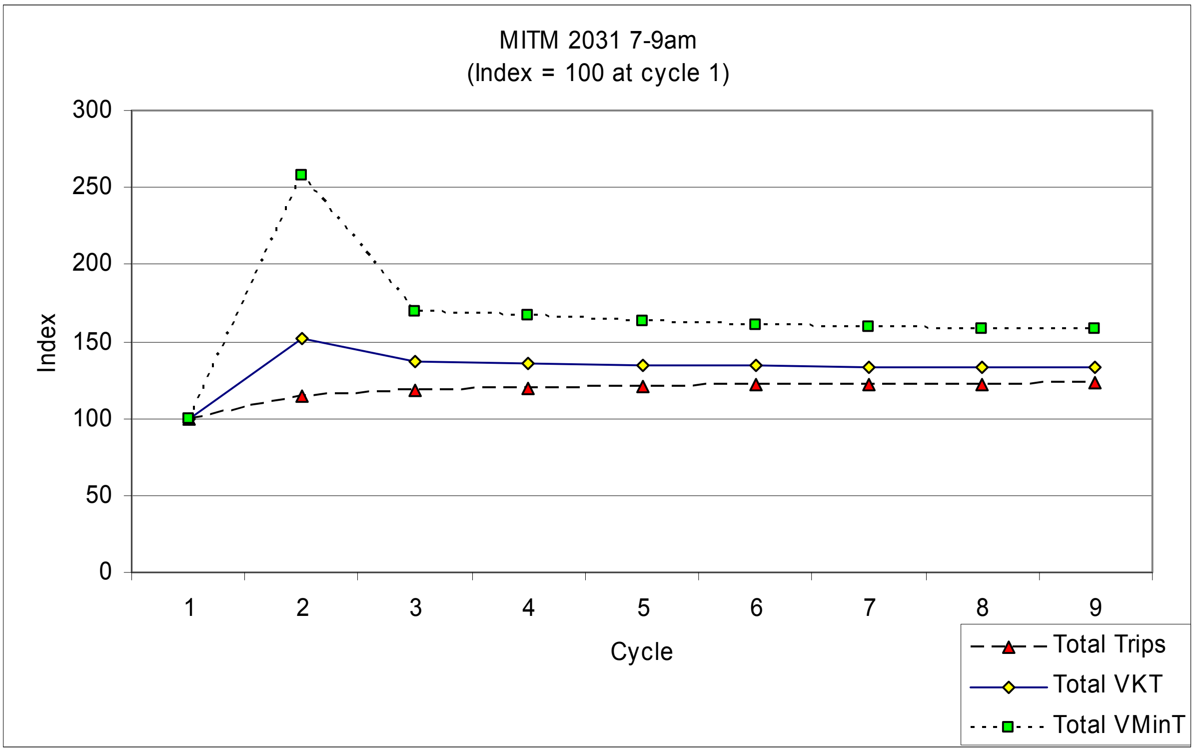

Figure 5 Convergent feedback. MITM 2031 (7-9am) with method of successive averaging. Results indexed to the value of the first cycle

The assigned traffic volumes for two major road links are shown by cycle number in Table 5.

Table 5 Convergent feedback. Assigned link volumes (7-9am; thousands of vehicles) by cycle for two major road sections in Melbourne for 2031

Cycle	Westgate Bridge (inbound)	CityLink Domain Tunnel (inbound)
1	17.9	5.1
2	19.3	6.9
3	18.5	6.9
4	18.4	6.9
5	18.3	7.0
6	18.3	7.0
7	18.3	7.0
8	18.2	7.0
9	18.2	7.0

The results in Tables 4 and 5 show that the change in the model results between cycles decreases as the number of cycles increases. Because MSA flow averaging is a convergent solution procedure, the solution after several cycles will approximate the correct solution. The magnitude of the difference between cycles shows that for MITM, six or seven cycles should provide stable results for most applications.

5 Test of the robustness of the solution method

The convergent solution procedure described above should be independent of the quality of the initial cost matrix. To test the above statement, an extreme example of an initial cost matrix was used within the feedback procedure. This involved the use of a matrix with equal cost for each and every zone to zone element. The value for each zone to zone element was chosen such that the sum of all the matrix elements was the same in the original and substituted matrix. In practice, such a "flat cost" matrix implies that the cost of travelling from one side of Melbourne to the other is the same as for travelling to adjacent transport zones. Such a test is considered to represent an extreme test of the robustness of the solution method implemented in MITM.

The total number of highway trips, total vehicle kilometres travelled and total vehicle minutes travelled are shown by cycle in Table 6 for the case of a flat initial cost matrix.

Table 6 Convergent feedback using initial flat-cost matrix. MITM model results, 2031 (7-9am period)

		Interzonal Trips Only		
Cycle	Terminating δ (%)	Total Veh. Trips (Million)	Total VKT (Million)	Total VMinT (Million)
1	2.3	1.51	62.0	3616.1
2	0.0	1.06	32.8	366.1
3	0.0	1.12	25.8	90.4
4	0.3	1.21	23.8	57.0
5	0.4	1.45	19.4	30.2
6	0.5	1.30	22.2	42.2
7	0.4	1.33	21.8	39.6
8	0.5	1.35	21.5	38.0
9	0.4	1.37	21.3	36.8
10	0.4	1.38	21.1	36.0
11	0.5	1.40	20.9	35.3
12	0.5	1.40	20.8	34.8
13	0.6	1.41	20.7	34.4
14	0.4	1.42	20.6	34.0
15	0.5	1.42	20.5	33.7
16	0.4	1.43	20.5	33.5

The interzonal data in Table 6 are plotted in Figure 6 with the results indexed to the value at the first cycle.

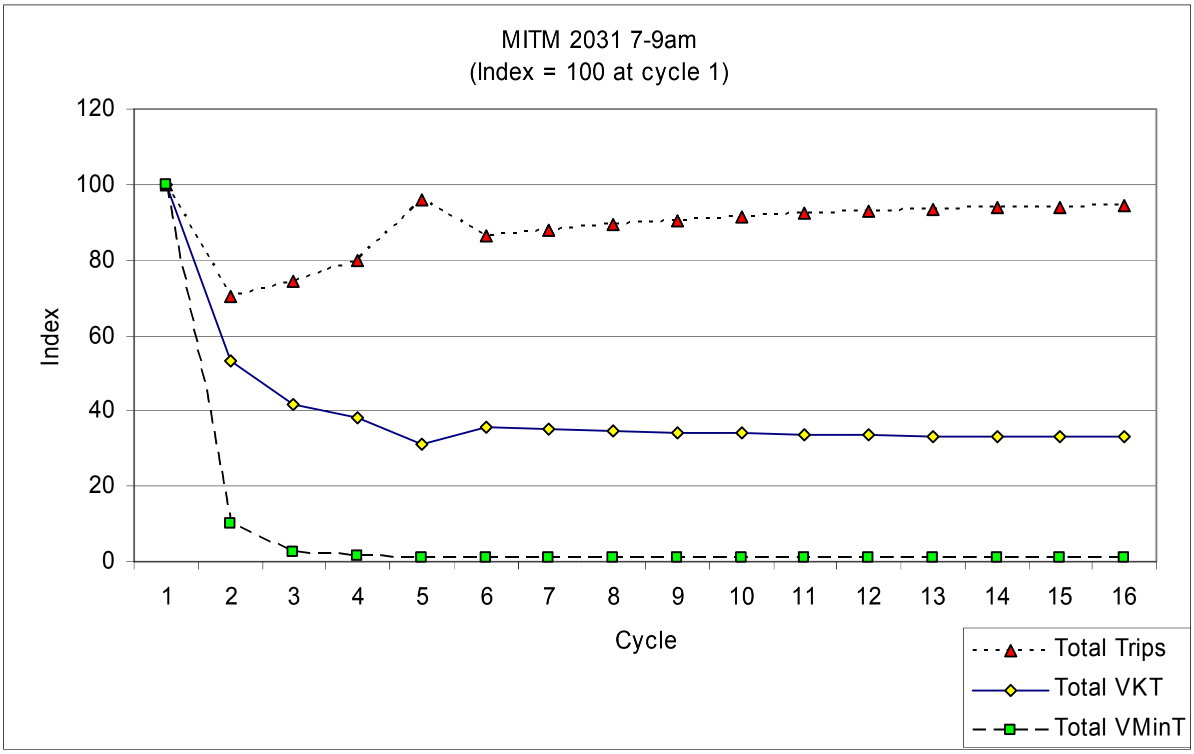

Figure 6 Convergent feedback using initial flat-cost matrix. MITM 2031 (7-9am). Results indexed to the value of the first cycle

The assigned traffic volumes for two major road links are shown by cycle number in Table 7 for the case of an initial flat-cost matrix.

Table 7 Convergent feedback using initial flat-cost matrix. Assigned link volumes (7-9am; thousands of vehicles) by cycle for two major road sections in Melbourne for 2031

Cycle	Westgate Bridge (inbound)	Citylink Domain Tunnel (inbound)
1	62.7	25.8
2	33.9	13.2
3	24.5	9.0
4	22.4	8.0
5	21.2	7.8
6	20.6	7.7
7	20.1	7.6
8	19.8	7.6
9	19.6	7.5
10	19.4	7.5
11	19.2	7.5
12	19.1	7.4
13	19.0	7.4
14	18.9	7.4
15	18.9	7.4
16	18.8	7.3

The results in Tables 6 and 7 show that even in the extreme case of using an initial flat-cost matrix, the procedure appears to approach similar results (within approx. 5%) to those using the original cost matrix (see Tables 4 and 5). This result provides a reasonable level of confidence that the convergent solution procedure described above is independent of the quality of the initial cost matrix.

6 Stopping criteria

The iterative procedure described above is guaranteed to eventually reach the correct unique solution to the combined distribution – mode split – assignment problem. Reaching this unique solution, however, may not be possible in a reasonable amount of time. In practice it is therefore necessary to specify criteria for when the iterative procedure should be stopped. These criteria should be chosen so that the difference between the fully converged solution and the result after the last cycle is acceptable. Difficulty arises because the fully converged solution is, of course, unknown.

Stopping criteria can be categorised into two groups: backward looking and forward looking. Backward looking criteria determine how much the results of the current cycle differ from the previous cycle. The iterative procedure is stopped when these differences become sufficiently small. The rationale is that if subsequent cycles do not change the result very much, then little is gained by continuing. However, how close the last cycle is to the fully converged solution is unknown.

Forward looking criteria, by contrast, use properties of the solution method to place a lower bound on the value of the objective function (which is minimised when the fully converged solution is reached). The iterative procedure is stopped when the result is guaranteed to be within a certain percentage of the fully converged result. See Rose *et. al.* (1988) for a general discussion of stopping criteria.

Backward looking stopping criteria are used in this work because the authors are unaware of appropriate forward stopping criteria for the combined distribution – mode split – assignment problem. Any model result that completely defines the solution and remains constant at equilibrium can be used as a backward looking stopping criterion. The cost matrix, for example, is a result that completely defines the solution and which should remain constant at equilibrium. If the costs remain constant, then the distribution of trips will be constant, as will be the mode split and assigned volumes.

Table 8 shows the variation in a number of potential stopping criteria by cycle for MITM with highway flow averaging using an initial flat-cost matrix for 2031. The stopping criteria reported by cycle are:

- The Percent Root Mean Square Error (%RMSE) for zone to zone travel times, where *%RMSE* is defined as:

$$\%RMSE_n = 100N \frac{\sqrt{\sum (V_n - V_{n-1})^2 / (N-1)}}{\sum V_{n-1}} \tag{2}$$

where N is the total number of components of a model variable (e.g. travel time, road link volume)
n is the individual component number
Σ is the summation of components (from 1 to N)
V is the value of an individual component (travel time, road link volume)

The %RMSE statistic provides a good indication of the percentage difference of parameters from one cycle to the next. For example, a %RMSE of 5 indicates that about two thirds of the individual zone to zone travel times are within 5% of those from the previous cycle.

Although usually the %RMSE statistic is used to measure the error of an approximation to the known true value of a parameter, here it is used to measure the difference between parameters calculated at subsequent cycles. This in no-way diminishes the usefulness of the statistic for these purposes.

- The %RMSE, or alternatively the maximum GEH statistic[2], for road link flows. The GEH statistic is defined as:

$$GEH_n = \sqrt{\frac{(V_n - V_{n-1})^2}{\frac{1}{2}(V_n + V_{n-1})}} \qquad (3)$$

with the maximum GEH statistic over all road links defined as:

$$\text{Max}\ (GEH_n); \qquad n = 1 \text{ to } N \qquad (4)$$

The GEH statistic provides a good indication of whether volumes from cycle to cycle are within acceptable percentage and absolute difference thresholds. A given GEH statistic permits a greater percentage difference threshold on lower volume links. The percentage and absolute difference for a given GEH and link volume are given by the equations:

$$abs = GEH \times \sqrt{LF} \qquad (5a)$$

$$\% = 100 \times GEH / \sqrt{LF} \qquad (5b)$$

where *abs* is the absolute difference; equivalent to $|V_n - V_{n-1}|$

% is the percentage difference; equivalent to $\frac{100 \times |V_n - V_{n-1}|}{\frac{1}{2}(V_n + V_{n-1})}$

LF is the link flow defined as $\frac{1}{2}(V_n + V_{n-1})$.

In the context of this work, GEH is not restricted to 1-hr volumes, as commonly used in the literature (see UK Highways Agency 1996), but may be applied to data for any time interval.

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[2] Named after Geoff E. Havers of Greater London Council

Table 8 Convergent feedback using initial flat-cost matrix. Stopping criteria for MITM (2031)

Cycle	%RMSE of hwy time OD pairs	%RMSE of hwy link flows	Max GEH of highway link flows
1	-	-	-
2	111.3%	80.3%	131.26
3	78.8%	41.9%	54.75
4	20.6%	15.2%	22.23
5	9.2%	8.1%	13.38
6	4.7%	5.2%	8.58
7	2.9%	3.7%	6.57
8	2.4%	2.7%	5.24
9	1.7%	2.1%	4.28
10	1.4%	1.7%	3.57
11	2.4%	1.4%	3.04
12	1.1%	1.2%	2.63
13	1.1%	1.0%	2.28
14	1.4%	0.8%	2.02
15	0.8%	0.7%	1.78
16	0.7%	0.6%	1.6

For results which are sufficiently stable for most transport planning applications, it is the authors' opinion that either of the following stopping criteria may be used:

%RMSE statistics for travel time **and** link flows	< 1%
Max GEH for link flows	< 2%

Whilst the GEH criterion proposed above applies to link flows, rather than travel time, a test on the maximum deviation of all individual links is intrinsically related to individual link-based travel times. For this reason the GEH criterion for link flows is considered to be a suitable alternative to a formal test of input and output travel times within each cycle.

In this work, both stopping criteria were satisfied by the fifteenth cycle as seen from Table 8.

The above stopping criteria are based on what the authors consider to be achievable within a reasonable amount of time for a large Australian city using commonly available desktop computers. Each MITM cycle takes 3-4 hours to run on a Pentium 4 PC running at 2.8 GHz with 2 GB of RAM. Incorporating the averaging step increases the time negligibly (by a few minutes per cycle). With improvements in computing power, or greater time resources, even more stringent criteria may be employed.

Moreover, whilst the proposed feedback procedure may in principle be used to evaluate each and every project scenario regardless of scale, the authors do not consider such an approach to be warranted. In other words, the straightforward assignment of an OD matrix (that has previously been processed via a convergent feedback procedure) to a highway network is considered to be reasonable for infrastructure projects that do not involve substantive network change(s). However, in cases where an OD matrix is established for a new year, or for major infrastructure projects that are anticipated to result in significant re-distribution of trips, the convergent feedback procedure is recommended every time.

7 Conclusion

The importance of achieving convergence in four-step transport models relates to the need to provide stable, consistent and robust model results. When model outputs are being used to compare the ‘base’ and ‘option’ scenario for a given infrastructure project (and subsequently to estimate the economic cost-benefit of the project), it is important to be able to distinguish real differences from those associated with different degrees of model convergence and spurious mathematical effects. Similar considerations apply when the benefits of different projects are being compared. Model convergence is therefore a key to robust transport planning and economic appraisal.

A transparent and convergent feedback procedure has been documented and implemented for the Melbourne Integrated Transport Model. The procedure, which includes the averaging of link flows and testing for convergence between cycles, is shown to produce more robust estimates of link volumes, travel times and vehicle kilometres of travel for use in the planning and economic assessment of infrastructure projects.

The procedure documented in this work is considered suitable for implementation in transport models for other major cities around Australia. However, VicRoads continues to refine and extend its processes for implementing feedback in four step transport models.

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