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# Australian Airport Network Robustness Analysis: A Complex Network Approach

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## Abstract

An airport network is a fundamental component of an air transportation infrastructure and has to be designed in such a manner that failure of any of its arbitrary components should not cascade into a catastrophic event. Resilience analysis of the Airports Network can offer an insight into airport network readiness and response behaviour to any catastrophic event. A complex network analysis approach of such a network can offer an insight about its structure, performance and resilience under various levels of perturbation.

In this paper, we present a complex network approach for measuring the performance and estimating the resilience of an airport network using the Australian Airports Network (AAN) as a case study. Real air traffic data for all domestic flights in the Australian airspace for the year 2011 is used to form the network. Resilience is then assessed under different failure scenarios. The complex network analysis reveals that the AAN can be classified as a scale-free small-world network and that its structure is resilient to random failures of airports (e.g., shutdowns) and random disruptions of flight paths (e.g., airways unavailable due to bad weather). It also indicates that the AAN remains connected and incurs minimal increase in travel times and reduced 'reachability' when a majority of its nodes (airports) are removed or its edges (airways) become randomly unavailable. In the case of a targeted failure (a targeted isolated airport shutdown), the AAN is more sensitive to node failure by a descending order of degree as well as 'betweenness centrality'.

## 1. Introduction

An air transport system and, more generally, any transportation infrastructure, plays a strategic role in our society in terms of both its economic significance and social impact. Air transportation is a principal means of the fast and effective movements of people and goods over large distances across countries and around the world, and is critical to the functioning of countries and the world economy as a whole.

Building an aggregate network of airports considering all flights among all destination airports throughout a country and the globe (the world airline network (WAN)) has been the subject of a great deal of recent complex network research (Amaral, Scala et al. 2000; Guimera and Amaral 2004; Bagler 2008; Wang, Mo et al. 2011). Network analysis to characterise complex systems has become widespread during the last few decades while complex network frameworks have been applied in a growing range of disciplines, such as technology (Huberman, Pirolli et al. 1998), artificial intelligence (Hossain 2010; Hossain, Abbass et al. 2010), biology (Yeger-Lotem, Sattath et al. 2004) and sociology (Castellano, Fortunato et al. 2009). More recently, the advancement of complex network theory has generated a huge interest in the area of applications in airport network systems (Guimera and Amaral 2004; Guimera, Mossa et al. 2005; Bagler 2008; Wang, Mo et al. 2011) in which the focus has been on analysing the overall network features and flow patterns, as well as identifying the importance of individual airports (Guimera, Mossa et al. 2005).

In complex network theory, an airport network is modelled as a graph (network), comprising the airports as vertices or nodes linked by flights connecting them. Interestingly, many real networks, including airport networks, share a certain number of topological properties; for example, most are small worlds (Amaral, Scala et al. 2000; Guimera, Mossa et al. 2005), that is, the average topological distances between nodes increase very slowly (logarithmically or even more slowly) with increases in the number of nodes. Additionally, ‘hubs’ (nodes with very large degrees (*k*) compared with the mean of the degree distribution ($P(k)$) are often encountered. More precisely, in many cases, the degree distributions exhibit heavy tails which are often well approximated for a significant range of values of $k$ by a power-law behaviour *(*$P(k) \sim k^{-\gamma}$*)* (Albert and Barabási 2002; Dorogovtsev and Mendes 2003). Although these topological features are extremely relevant for characterising network topology, research is needed to identify the topological features required to assess an air transportation network’s robustness or vulnerability. This is important and has been linked to events such as 9/11, natural disasters such as Hurricane Katrina, the Icelandic volcano which erupted in 2010 and blocked air travel in a large area, fatal runway incursions (e.g., the Linate runway collision in 2001) and fatal mid-air collisions (e.g., over Ueberlingen in 2002). With the aim of hedging against natural disasters and hazardous events, a majority of the complex network literature focuses on the graph characteristics of the associated application to evaluate a network’s reliability and vulnerability (Holme, Kim et al. 2002; Chassin and Posse 2005).

However, in order to be able to evaluate the reliability and vulnerability of an airport network, a measure or set of measures that can quantifiably capture its efficiency/performance and resilience must be developed. To achieve this, we are interested in two main questions:

(i) which network measures are best suited for assessing the damage incurred by airport networks and characterising the most effective attack (protection) strategies; and

(ii) how does a network’s structure influence its/the system’s robustness?

Therefore, our attention is focused on network topology and the analysis of its structural vulnerability with respect to various centrality-driven failure (attack) strategies. In particular we propose a series of topological features that can be used to identify the most important node(s) of an airport network.

Although the topological structure of a network obviously has an impact on the network’s performance and vulnerability, we believe that the traffic flowing through such structure is also an important indicator, as are its induced costs and users’ behaviour. A network’s flows represent its usage and the paths and links with positive flows, together with their magnitudes, are relevant in the case of network disruptions. Therefore, a network efficiency measure that captures flows and their associated costs along with topology is a higher resolution indicator for network resilience.

As a case study, we investigate the features and resilience under disruption of the Australian Airport Network (AAN). There are two main types of disruption that can occur in an airport network.

(i) Short-term outages of resources, such as airspace closures for special purposes (e.g., military use), unavailability of the runway and other airport facilities due to bad weather or congestion, limited operation of control tower radar due to servicing and breakdowns in communication links, are usually recovered within a short period of time and normally only slightly impact on the overall performance of the relevant systems. In the current air transportation system, if an airport is congested, all its departure flights are delayed and its expected arrival traffic delayed in the airports of origin. Airborne flights travelling to a malfunctioning airport are either put in a holding pattern if possible or diverted to another airport until the problem is rectified.

(ii) Long-term resource outages resulting from: events such as terrorist actions (e.g., 9/11 in 2001) or volcanic ash in the sky (e.g., following the volcanic eruption in Iceland in 2010) which cause the suspension of air travel over large areas; airports being affected by flooding; runway accidents that damage the runway; and airports being damaged by tornadoes or typhoons. These are rare events that cannot be recovered in a short period of time, and could trigger a catastrophic failure of entire systems.

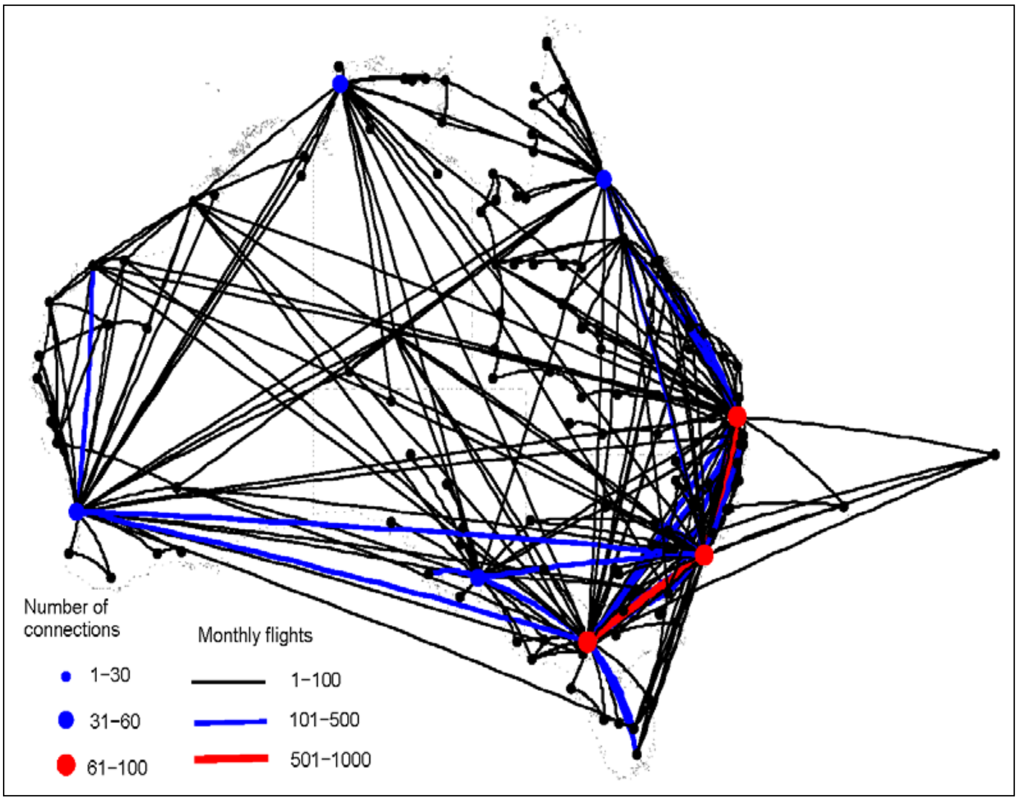
In this study, although we focus on the effect of a long-term disruption on the resilience of the AAN, the results and insights are applicable to other airport or transportation networks with similar features. We analyse the AAN's topological features and resilience by considering the intensity of its interactions in order to gain an insight into its features and characteristics which may help in the design of a more efficient air transport network with high resilient to node failures (airport shutdowns) and link disturbances (airspace disruptions or service closures).

The rest of this paper is structured as follows: in section 2, the model representing an airport network is defined as a graph; in section 3, the topological features characterising the AAN are described; in section 4, a resilience evaluation and several centrality-based failure (attack) strategies are presented; in section 5, the resilience and traffic re-routing costs are evaluated and analysed and finally the conclusions are drawn.

## 2. Network Modelling

The AAN consists of domestic and international airports for handling regular passenger flights conducted by more than 20 airlines (domestic and regional). The air movement data for Australian airport-pairs for the year 2011 was obtained from the Bureau of Infrastructure, Transport and Regional Economics, Australia (http://www.bitre.gov.au) and Official Airline Guide–OAG (http://www.oag.com).

**Figure 1: Spatial distributions of nodes and edges of AAN**



A graph ($G(V, E)$) is used to describe the AAN in which the node set ($V$) represents all $n$ airports and the edge set ($E$) all the $m$ direct flight routes between airports. The network is represented by an adjacency matrix ($A_{n \times n}$) such that $a_{ij} = 1$ if a flight link exists between the city-pair $i$ and $j$, otherwise $a_{ij} = 0$. From the results, we find that the AAN is a directed network in which all major airports have direct connections. For developing the network links, both direct and stop-over air routes are considered, with links created from a start to stop-over airport, from it to the next stop-over airport (if there is more than one stop-over) and then to the destination airport. Duplicate air routes are removed, with only the unique ones maintained between each airport-pair which results in the AAN consisting of $n = 131$ airports and $m = 596$ links (directed links).

As for other complex networks, the flow of information (traffic load) is a crucial factor for any transportation network and, to accommodate it, the AAN is represented as a weighted network by considering the number of flights flying on a route as the ‘weight’ of that particular link defined by a weight matrix ( $A^w$), where each element ($w_{ij}$) represents the total number of flights from airport $i$ to airport $j$. Figure 1 shows the weighted AAN in which the proportional circles represent the number of air connections of airports (number of routes) and the widths of the links the monthly average volume of traffic.

**Table 1: Top 10 Airports by number of flights in 2011**

| Rank | Airport | Number of connected routes | Flight total (in thousands) |
|---|---|---|---|
| 1 | Sydney | 99 | 158.212 |
| 2 | Melbourne | 66 | 149.196 |
| 3 | Brisbane | 81 | 110.209 |
| 4 | Perth | 57 | 58.352 |
| 5 | Adelaide | 47 | 47.584 |
| 6 | Gold Coast | 25 | 31.711 |
| 7 | Cairns | 46 | 24.829 |
| 8 | Canberra | 24 | 31.011 |
| 9 | Hobart | 10 | 14.761 |
| 10 | Darwin | 41 | 11.321 |

Table 1 summarises the air-traffic volumes and routes (numbers of links connected with other airports) of the top 10 cities of the AAN from January 2011 to December 2011. The air-route data includes all the airlines (domestic and regional) that provide connectivity between the airport-pairs and, of the 131 airports, Sydney has the highest number of both air-route connections and flight movements.

## 3. Network Features and Definition

Network structures occur in a wide range of different contexts, such as technological and transportation infrastructures, social phenomena and biological systems, and each class presents specific topological features which characterise the connectivity, interaction and dynamical processes executed by each type of network (Barrat, Barthelemy et al. 2004). Therefore, the analysis, discrimination and synthesis of a complex network relies on the use of measurements capable of expressing the most relevant topological features which enable us to characterise its complex statistical properties (Costa, Rodrigues et al. 2007). Several basic indices are used in this study to characterise the topology of the AAN and its robustness.

**Degree Distribution:** The degree is the most important characteristic of a vertex or node and is the number of edges a node shares with others (Barabási and Albert 1999), with that of node *i* defined as:

$$k_i = \sum_{j=1}^{n} a_{ij}$$

The average degree of a network is the average number of neighbours a node has which is denoted as $\langle k \rangle$ and written as:

$$\langle \mathrm{k} \rangle = \frac{1}{n} \sum_{i=1}^{n} k_i$$

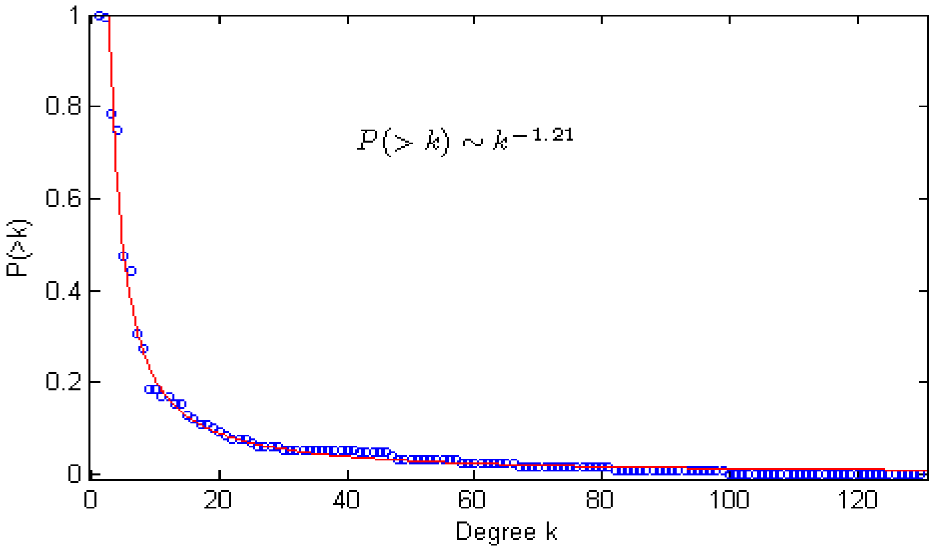
The value of its degree symbolises the importance of a node in a network – the larger it is, the more important the node – and, based on the degrees of its vertices, it is possible to derive important measures for the network, including its degree distribution. For a network with $n$ nodes, if $n_k$ of them have a degree of $k$, the degree distribution ($P(k)$) is defined as the fraction of these $k$-degree nodes, i.e., ${}^{n_k}/_{n}$. $P(> k)$ represents the cumulative degree distribution, i.e., the fraction of nodes with degrees greater than or equal to $k$ and is formulated as:

$$P(> k) = \sum_{k'=k}^{\infty} P(k')$$

The distribution of degrees in a network is an important feature which reflects the topology of the network and sheds light on the process by which it came into existence. A random network (an Erdös-Renyi graph) shows a Poisson (exponential) connectivity distribution ($P(k)$) (the probability that a node has degree *k*) that peaks at $\langle k \rangle$ and decays exponentially for large $k$, and no nodes play any type of central role in it. As, for a scale-free network, $P(k)$ decays as a power law ($P(k) \sim k^{-\gamma}$), it may be described as being free of any single, characteristic scale (Albert and Barabási 2002).

In a scale-free network, it is likely that more of its connections will use a hub, a dependence which is an important factor governing the behaviour of the two types of network under degradation, e.g., when and how hubs fail. Thus, while certain scale-free networks may display high levels of resilience to degradation (due to failure or attack), others may be more sensitive than a random network if the hubs are more likely to fail. Although the power-law distribution implies that nodes with smaller connectivities will be affected by much higher probabilities of random disruption, this cannot be assumed to be the case with air transport hubs.

**Figure 2: Cumulative degree distribution of AAN**



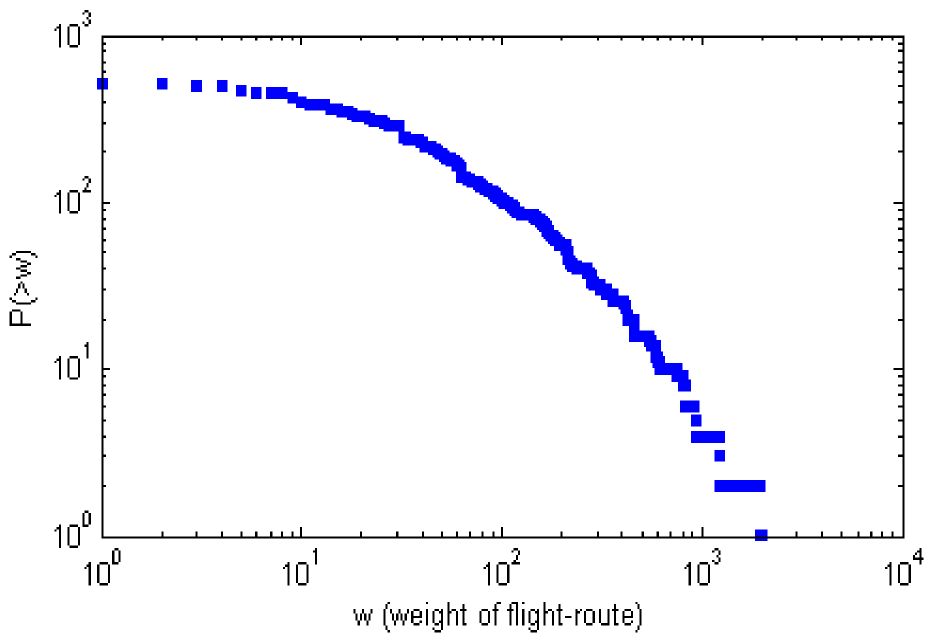
The cumulative degree distribution ($P(>k)$) of the AAN is illustrated in Figure 2 in which it can be seen that it follows a power-law distribution $P(>k)\sim k^{-1.211}$, with a wide range of degrees ($k_i$), which confirms that a small number of nodes carry the majority of the routes. Indeed, as the top 10 most-connected airports account for 43% of the air routes, this scale-free structure could be robust to a random failure but vulnerable to a targeted or specific failure.

**Weight and Strength:** The weighted counterpart of degree is called strength ($S_i$) which is the total load on all its links (Barrat, Barthelemy et al. 2004) and is formulated as:

$$S_i = \sum_{j=1}^{n} a_{ij} w_{ij}$$

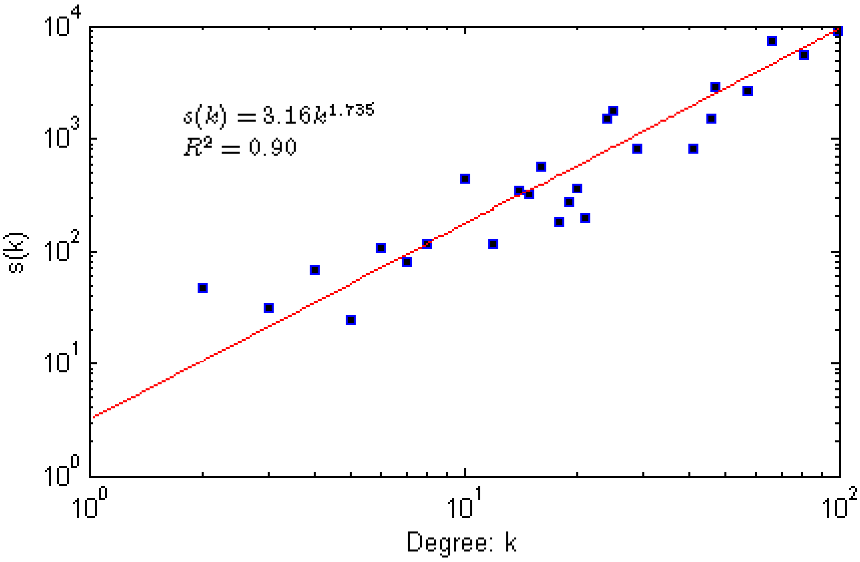
The cumulative weight distribution ($P(>w)$) of the AAN is plotted in Figure 3 in a double-logarithmic scale. The statistical analysis of the weights between pairs of airports ($w_{ij}$) indicates the presence of right-skewed distributions which signal a high level of heterogeneity in the system,a phenomenon also found in the case of the airport networks of India (Bagler 2008) and China (Wang, Mo et al. 2011), and the WAN (Guimera, Mossa et al. 2005).

**Figure 3: Cumulative weight distribution of AAN**




It has been observed that the weights of its individual links do not provide a general picture of a network's complexity (Yook, Jeong et al. 2001) and a more significant measure of this considering the flow of information could be its node strengths ($S_i$). This parameter measures the strength of a node in terms of the total weight of its connections and is a natural measure of the node's importance or complexity in the network. To capture the relationship between a node's strength and degree, we investigate the dependency of $S_i$ on $k_i$, as shown in Figure 4 for the average strengths of nodes with degree $k$ ($s(k)$). We find that, in the AAN, $s(k)$ increases with increasing degrees as $s(k) \sim k^{\beta=1.735}$ while the value of $\beta = 1.735$ implies that the nodes' strengths are strongly correlated with their degrees, an expected behaviour because it is plausible that the larger an airport in terms of connections, the more traffic it handles.

**Figure 4: Average strength $(s(k))$ as a function of degree $k$**




**Clustering coefficient**: The clustering coefficient ($C_i$) of a node ( $i$ ) is defined as the ratio of the number of links it shares with its neighbouring nodes up to the maximum possible number. In other words, $C_i$ is the probability that two nodes are linked to each other given that they are both connected to $i$ (Newman 2001) and, for node $i$ is formulated as:

$$C_i = \frac{1}{k_i(k_i - 1)} \sum_{i,j,k} a_{ij} a_{jk} a_{ik}$$

In a network, a large value of $C_i$, means that node $i$ has a more compact system of connections with its neighbour and, in a fully connected network, the $C_i$ of all the nodes equal 1. On the other hand, if a node has only one neighbour, i.e., a degree of 1, its $C_i$ equals 0. The clustering coefficient of the overall network ($C$) is the average of all individual $C_i$, and mathematically represented as:

$$C = \frac{1}{n} \sum_i C_i$$

The AAN's $C = 0.50$ which is much larger than that of a random network ($C_{rw} = 0.091$) of the same size and similar to that of a small-world network ($C_{sw} = 0.635$). This confirms the AAN's high degree of concentration and also implies a high probability of travelling in it with fewer stop over.

**Characteristics Path Length ($L$)**: The characteristics path length of a network is defined as the average number of edges along the shortest paths for all its possible node-pairs (Watts and Strogatz 1998) which is also known as the average shortest path length and written as:

$$L = \frac{1}{n(n-1)} \sum_{v \in V} \sum_{w \neq v \in V} d(v, w)$$

where $d(v, w)$ is the length of the geodesic between $v$ and $w$ $(v, w \in V)$, i.e., the number of edges in the shortest path connecting the two, and the factor $n(n-1)$ the number of pairs of nodes. From the geodesic, one important property of a network, the 'diameter', is defined as the maximum value of $d(v, w)$ of all node-pairs.

In a weighted network, unequal link capacities make some specific paths more favourable than others for connecting two of its nodes. A passenger's choice of route depends roughly on the geographical distance of, and the number of flights operating on a route. A straightforward way of generalising the hop distance in a weighted network should include the number of flights and physical distance between airports $i$ and $j$ ( $w_{ij}$ and $d_{ij}$, respectively). It is quite obvious that the effective distance between two connected nodes is a decreasing function of the weight (traffic) of the link. In others words, the larger the traffic flow on a route, the more frequent and faster will be the exchange of physical quantities, and it also reflects users' choice. Therefore, we define the effective distance of a link (route) as a ratio of $d_{ij}$ to $w_{ij}$ and, in this paper, represent this measure as $\varphi$ and define it as:

$$\varphi_{ij} = \begin{cases} \frac{d_{ij}}{w_{ij}} & if\ a_{ij} = 1 \\ \infty & otherwise \end{cases}$$

To generalise the notation of the characteristics path length through a weighted network, especially an air transportation network, we propose that the shortest paths defined by the hop distance be replaced by the effective distance of an edge ($\varphi_{ij}$) which can be mathematically formulated as:

$$L^{\varphi} = \frac{1}{n(n-1)} \sum_{v \in V} \sum_{w \neq v \in V} d^{\varphi}(v, w),$$

where $d^{\varphi}(v, w)$ is the shortest path of effective distance between $v$ and $w$.

**Table 2: Characteristics of air transport network of Australia and other countries/regions**

| Author | Country | No. of nodes (n) | No. of edges | ⟨k⟩ | L | C | Network structure |
|---|---|---|---|---|---|---|---|
| Bagler | India | 79 | 455 | 11.52 | 2.26 | 0.66 | SW |
| Guimer´a et al. | World | 3883 | 27051 | 13.93 | 4.4 | 0.62 | SF SW |
| Guida and Maria | Italy | 50 | 310 | 12.4 | 1.98~2.14 | 0.07~0.1 | SF SW Fractal |
| Xu and Harriss | US | 272 | 6566 | 48.28 | 1.84~1.93 | 0.73~0.78 | SW |
| J. Wang et al. | China | 144 | 1018 | 14.14 | 2.23 | 0.69 | SW |
| In this paper | Australia | 131 | 596 | 9.10 | 2.90 | 0.50 | SW |

Table 2 compares the topological properties of the AAN with those of other similar types of air transport networks. The average shortest path length ($L$) of the AAN which is 2.90, implies that, on average, it requires almost three flight changes to connect most city-pairs, and is slightly larger than those of China (2.23) and India (2.26) but much larger than that of the US (which ranges from 1.84 to 1.93). As, according to Watts and Strogatz (1998), if $L$ increases almost as fast as $log(n)$, where $n$ is the number of nodes, the corresponding network can be defined as a small-world network, from this analysis, it can be inferred that the AAN has evolved as a **small-world** topology which means that it might be possible to connect from one node to another through a small number of nodes (even in the absence of hubs); in particular, for the AAN, $L = 2.90$ and $log(n) = 2.12$ for $n = 131$. As this relatively larger value of $L$ implies that more flight stops are needed to connect any two cities in the AAN, there is a great deal of room to improve its efficiency in terms of connections.

The average degree of the AAN of $\langle k \rangle = 9.10$ is the lowest of the networks considered in this paper whereas its clustering coefficient ($C = 0.5$) is slightly smaller than those of India (0.66) and China (0.69) but much smaller than that of the US (0.73-0.78). All the network features presented in Table 2 confirm that the AAN has properties similar to small-world characteristics.

### 3.1. Centrality Measures

A key issue in the characterisation of a network is identification of its most important nodes which can be achieved through the concept of centrality which can be quantified by various measures. The degree ($k_i$) is the first intuitive parameter that gives an idea of the importance of a node in terms of connectivity while the strength ( $S_i$,) takes its amount of traffic (operating load) into account.

However, these local measures do not consider non-local effects, such as the existence of bottleneck nodes which may have small degrees but act as bridges between different parts of a network. In this context, a well-accepted parameter for investigating node centrality is the 'betweenness centrality' (Freeman 1977) which, for a node ( $i$) is defined as the ratio of all the shortest paths passing through it and reflects its transitivity. More precisely, if $\sigma_{kj}$ is the total number of shortest paths from node (vertex) $k$ to $j$ and $\sigma_{kj}(i)$ the number of them that pass through node $i$, the betweenness centrality of node $i$ is defined as:

$$B_v(i) = \sum_{k \neq j \neq i} \frac{\sigma_{kj}(i)}{\sigma_{kj}}$$

The betweenness of a network ($B_v$) is the average of the node betweenness of all its nodes.

**Figure 5: Betweenness centrality distribution of AAN**

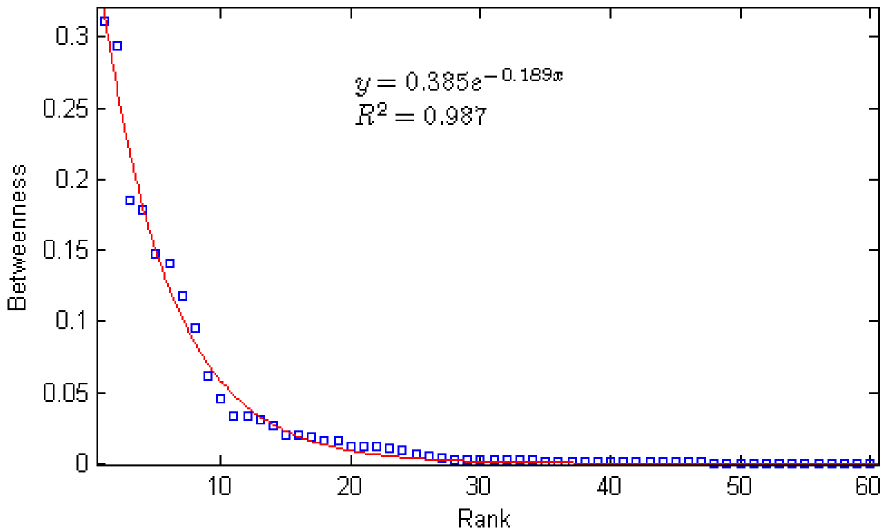



Figure 5 shows the distribution of the node betweenness centrality of the AAN which shows that 118 nodes (around 90% of the total number) have less than average betweenness values ($B_v = 0.0145$). The sharp decline in the betweenness curve and the large number of nodes with low betweenness values suggest the existence of bottlenecks within the AAN, as confirmed by its low clustering coefficient ($C = 0.5$).

Similar to the importance of the most central nodes in a network, the most weighted links are also significant, especially in a transportation network. As suggested by (Newman and Girvan 2004), the links with the highest betweenness values are most likely to lie between sub-graphs rather than inside a sub-graph. Consequently, successively removing the link (edge) with the highest edge-betweenness value will eventually fragment a network consisting of nodes that share connections with only the other nodes in the same sub-network. The edge-betweenness ($B_e$) for edge *m* is defined as:

$$B_e(m) = \sum_{k \neq j} \frac{\sigma_{kj}(m)}{\sigma_{kj}}$$

where $\sigma_{kj}(m)$ is the total number of shortest path edges between nodes *k* and *j* that include the edge *m*.

**Table 3: Top 20 Airports by degree and betweenness centrality**

| Rank | Degree | Betweenness |
|---|---|---|
| 1 | Sydney | Brisbane |
| 2 | Brisbane | Sydney |
| 3 | Melbourne | Cairns |
| 4 | Perth | Perth |
| 5 | Adelaide | Adelaide |
| 6 | Cairns | Darwin |
| 7 | Darwin | Melbourne |
| 8 | Townsville | Mount Isa |
| 9 | Gold Coast | Townsville |
| 10 | Canberra | Toowoomba |
| 11 | Broome | Launceston |
| 12 | Avalon | Charleville |
| 13 | Alice Springs | St. George |
| 14 | Mount Isa | Boulia |
| 15 | Launceston | Gold Coast |
| 16 | Newcastle | Avalon |
| 17 | Karratha | Quilpie |
| 18 | Mackay | Doomadgee Mission |
| 19 | Rockhampton | Cunnamulla |
| 20 | Geraldton | Bedourie |

Table 3 lists the top 20 Australian cities, the most central nodes in the AAN, by degree and betweenness which shows that the same 13 are in both centrality indices. Brisbane is ranked at the top for betweenness and Sydney at the top for degree, followed by Brisbane. Melbourne is ranked 3rd for degree and 7th for betweenness which indicates a high level of inconsistency in the centrality indices. Perth and Adelaide rank in 4th and 5th places, respectively, in both indices. The national capital, Canberra, is ranked 10th for degree but is not in the top 20 for betweenness.

# 4. Resilience of Australian Airport Network

Resilience refers to the ability of a network to avoid malfunctioning when a fraction of its constituents are damaged. According to Eurocontrol (2009), “Resilience is the intrinsic ability of a system to adjust its functioning prior to, during, or following changes and disturbances, so that it can sustain required operations under both expected and unexpected conditions.” Thus, the resilience analysis of a transportation network is very important to the understanding of its sustainability as it directly affects the efficiency of any process running on top of the network and is one of the important issues being explored in the literature (Newth and Ash 2004; Boccaletti, Latora et al. 2006; Wuellner, Roy et al. 2010). As the use of multiple metrics and simulation provides a promising approach for addressing the complexity of resilience, this paper measures an airport network’s resilience through evaluating its topological and reachability metrics.

## 4.1. Metrics for Damage Characterization

To analysis the sensitivity of the AAN to various failure scenarios, we primarily use two types of measures: (i) topological sensitivity; and (ii) reachability and re-routing cost.

#### 4.1.1. Topological Sensitivity

There are several ways of measuring the topological sensitivity of a network, with one key parameter, the average geodesic path length ( $\boldsymbol{L}$). As the number of removed nodes or links increases—leading to the network eventually breaking into disconnected sub-networks (sub-graphs)—$\boldsymbol{L}$ will approach infinity for such a disconnected graph. It is then wise to study the average inverse geodesic length as:

$$\ell^{-1} = \frac{1}{n(n-1)} \sum_{v \in V} \sum_{w \neq v \in V} \frac{1}{d(v,w)},$$

which is a finite quantity even for a disconnected graph since $1/d(v,w) = 0$ when there is no path connecting $v$ and $w$.

For many applications, the distance between a pair of nodes in the network is one of the most important determinants of network efficiency. When nodes are separated by short effective distances, they can easily communicate and distribute resources to each other. This idea motivated the following 'distance-attenuated reach' metric that includes the effective distance to the average inverse geodesic length.

$$(\ell^{\varphi})^{-1} = \frac{1}{n(n-1)} \sum_{v \in V} \sum_{w \neq v \in V} \frac{1}{d^{\varphi}(v,w)}$$

Since subsequent failures of nodes or links might fragment a network, two important quantities for measuring its sensitivity are the number of sub-graphs and size of the largest connected sub-graph (giant component) which, in this paper, are represented by $N_c$ and $S$**,** respectively. In order to quantify the effect of any node or link failure, the values of the average inverse geodesic length, distance-attenuated reach, number of sub-graphs and giant component are normalised by the corresponding values of the initial or original network and represented by $\widetilde{\ell^{-1}}, \widetilde{(\ell^{\varphi})^{-1}},\ \widetilde{N_c}\ and\ \widetilde{S}$ respectively.

#### 4.1.2. Reachability and Re-routing cost

If a node or link is removed from a directed network, this raises the question as to whether the network is fully reachable, that is, starting from any node, is it possible to reach all other nodes in the network?. In order to assess the reachability of the AAN, we calculate the probability of the connectivity between any pair of its nodes,$(v,w)$, which is represented by $R$, and the reachability of node $R_i$ is calculated as:

$$R_i = \frac{number\ of\ nodes\ reachable\ from\ node\ i}{n-1}$$

The reachability ($R$) of the overall network is defined as the average of all $R_i$, with a fully reachable network achieving 1 and an isolated network with no physical connection (links) between the nodes is always 0.

We turn our attention to traffic flows following the failure of some of a network's elements. If a node is disconnected from the network, the traffic associated with it raises the question of related damage costs (traffic re-routing) as, after a node failure, the traffic destined for that airport needs to be re-routed to the nearest possible airport. But this raises the issue of whether or not the nearest airport is capable of handling any types of aircraft (light, medium and heavy)? Since some airports are incapable of handling heavy and medium aircraft, flights must be re-routed to the closest appropriate airport. For example, if Sydney suffers an operational failure, all the light and medium aircraft could be re-directed to Newcastle airport and the heavy aircraft re-routed to Canberra. Thus Newcastle and Canberra could act as 'surrogate airports' for Sydney. This re-routing definitely incurs additional costs to airline operators, such as those of flights which might need to travel greater geographical distances as well as those associated with landing in surrogate airports. To estimate the re-routing cost of a node failure, we first define the operating cost ($c_{ij}$) of a city pair ($i, j$) as a product of the related geographical distance ($d_{ij}$) and traffic ($w_{ij}$) as $c_{ij} = w_{ij} d_{ij}$. Then, the re-routing cost for a node '$x$' failure ($RC(x)$) is calculated using the following algorithm.

**Algorithm I:** Re-routing cost estimation

$x$-failed node; $w_{ix}$ - weight (traffic) from node $i$ to node $x$

***for*** each source and destination to '$x$'-node pairs $(i, x)$
    ***if*** $w_{ix} \neq 0$ ***then***
        decompose light ($w_{ix}^{l}$), medium ($w_{ix}^{m}$), and heavy ($w_{ix}^{h}$) where $w_{ix}^{l} + w_{ix}^{m} + w_{ix}^{h} = w_{ix}$
        $sl$: surrogate node of $x$ that can handle light aircraft
        $rc^{l} \leftarrow (w_{ix}^{l} |P^{sl}| + c\, w_{ix}^{l} d_{sl\,x} - w_{ix}^{l} d_{ix})$ ; where $|P^{sl}|$ is the length of shortest path from $i$ to $sl$
        $sm =$ surrogate node of $x$ that can handle medium aircraft
        $rc^{m} \leftarrow [\![(w]\!]_{ix}^{m} |P^{sm}| + c\, w_{ix}^{m} d_{sm\,x} - w_{ix}^{m} d_{ix})$
        $sh$: surrogate node of $x$ that can handle heavy aircraft
        $rc^{h} \leftarrow (w_{ix}^{h} |P^{sh}| + c\, w_{ix}^{h} d_{sh\,x} - w_{ix}^{h} d_{ix})$ ;
        $rc_i \leftarrow rc^{l} + rc^{m} + rc^{h}$
    ***end if***
***end for***
$RC(x) = \sum rc_i$

For estimating the re-routing costs, besides that of re-routing the traffic to a surrogate node(s), there are other consequences, such as passengers needing to travel to their original destinations by other forms of transport which will be more expensive than the normal operating cost ($\sum_i w_{ix} d_{s\,x}$) of travelling from the surrogate to failed node. We consider that the extra cost is 'c' times $\sum_i w_{ix} d_{s\,x}$ which is added to the overall re-routing cost ( $RC(x)$) and set $c = 1.5$.

## 4.2. Failure Scenarios

In order to evaluate the vulnerability of an airport network, the selection procedure for the order in which nodes or links could be removed is an open choice. A node failure simply corresponds to the closure of an airport whereas a link (edge) failure corresponds to, for instance, disturbances such as weather or airspace closures which prevent travel between a pair of airports. We analyse the behaviour of damage measures in the presence of a progressive random damage and provide several attack strategies.

Transportation networks are inherently resilient to random node or edge failures and, even after a large number of them, all the metrics measures decrease only moderately and do not seem to reach a sharp threshold after that the network is virtually destroyed (Dall'Asta, Barrat et al. 2006). Since one of the objectives is to identify the nodes or edges which maximise disruption in a network, one approach is to select the most central nodes. A straightforward choice is to select the nodes (vertices) in a descending order of degrees of the initial network and then remove them one by one starting from that with the highest degree (Barabási and Albert 1999). In addition, we use various strategies based on the different definitions of centrality ranking of the most important node and a node can be removed according to its strength ($S_i$) and topological betweenness ( $B_v(i)$). Apart from a node failure, edge or link failures also play very important roles in analysing the resilience of a transportation network. Edge selection can be random or in a descending order of the weight (traffic) and edge betweenness ($B_e(m)$) of the original network.

# 5. Results and Analysis

## 5.1. Tolerance to Node Failure

Tolerance to errors (or random failures) is understood as the capability of a system to restore its structural properties after deletion of a fraction of its nodes or edges. At first, we simulate the failure vulnerability of the AAN under a randomly fractional node failure ($f_v$) and then according to the ranks of the degree ($k_i$), node betweenness ($B_v(i)$), strength ($S_i$) and φ-strength ($S_i^{\varphi}$) strategies.

**Figure 6: AAN resilience against node failure**

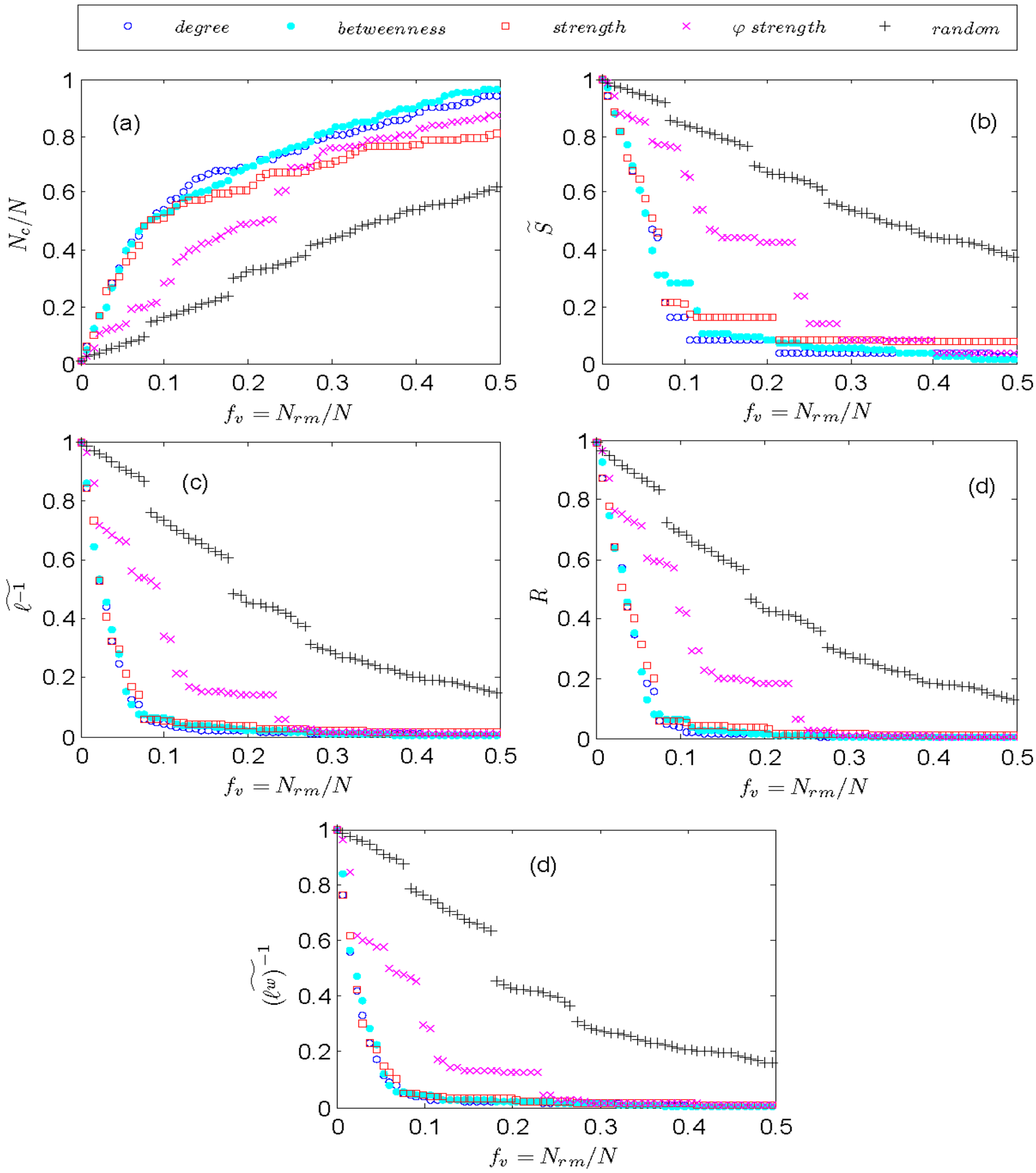



Figure 6, summarises the results obtained by the node failure measure of topological sensitivity – the number of sub-graphs $(N_c/N)$, relative size of the giant component $(\widetilde{S})$, average inverse geodesic distance $(\ell^{-1}(\text{topological}) \text{ and } (\ell^{\varphi})^{-1}\ (weighted))$ and network reachability $(R)$ as functions of the number of node failure ( $f_v = N_{rm}/N$).

Figure 6 (a) shows how the network fragments as the number of node failure ( $f_v = N_{rm}/N$) increases. Within the range of $50\%$ of the nodes failing (removed), the number of sub-graphs ($N_c$) increases more rapidly for a centrality-based node failure. In this measure, the degree- and betweenness-based failure strategies have almost the same network fragmentation effect.

Figure 6(b) illustrates the changes in size of the giant component with the increasing number of node failures. After removing about $10\%$ of the highly connected or high betweenness nodes, the size of the gain component (the largest sub-graph) reduces abruptly to 20% of the original size of the AAN, with similar behaviour observed for a strength-based failure. After removing 50% of the nodes, as the network becomes almost fragmented at that point, the size of the largest sub-graph is too small and, as a consequence, the network loses its functionality. However, for a random node failure, the relative number of sub-graphs ($N_c/N$) and size of the giant component ($\widetilde{\mathrm{S}}$) change linearly with $f_v$.

However, if we look at the network efficiency measured by the average inverse geodesic distance ($\ell^{-1}$ and $(\ell^{\varphi})^{-1}$) and network reachability ($R$), all these metrics behave almost identically regarding any method of node failure. For centrality-based failures (degree and betweenness), network efficiency drops abruptly at the very beginning of the process, with the values of $\ell^{-1}$, $(\ell^{\varphi})^{-1}$ and $R$ decreasing to almost zero just after 10% of the highly connected or central node breaks down. It implies that after removing a few hubs from the network, the probability of a pair of nodes becoming connected is very low, the geodesic distance between them increases and, as a result, the inverse geodesic distance ($\ell^{-1}$) becomes very small and triggers a high transportation cost.

On the other hand, in the case of a random failure, the network seems highly robust as the topological and reachability metrics change almost linearly with the number of node failures ( $f_v = N_{rm}/N$). From Figure 6, it can be summarised that the removal of a small proportion of the highly connected or central nodes produces catastrophic changes in the network topology and functionality. This analysis suggests that the AAN is highly vulnerable under centrally driven failure scenarios compared with random or unintentional failures.

## 5.2. Node Criticality

Up to this point in the comparison of vulnerability approaches, it is difficult to determine the failure of which node would be most damaging in terms of both topological sensitivity and re-routing cost. That said, an alternative approach, particularly a node-by-node failure approach for capturing the topological sensitivity and re-routing cost measures, provides a better summary of node criticality or importance. To measure the sensitivity of a node, we measure its topological metric by removing it from the network and comparing its resilience metric with its corresponding value in the original AAN. The re-routing cost for traffic due to the failure or shutdown of a node is calculated using Algorithm I described in section 4.1.2.

Figure 7 shows the route map of the original AAN (left side) and re-routing of the traffic (right side) if Sydney airport's operation fails (shutdown) in which case all flights to Sydney are re-routed to either Newcastle (light and medium aircraft) or Canberra (heavy aircraft) which act as Sydney's surrogate airports. In Figure 7, the blue lines represent the links where the numbers of flights (traffic) remain unchanged and the red line represents increase in the traffic. In this re-adjusted route map (right side), it is noticeable that the impact of Sydney airport being disconnected from the network causes a large number of traffic re-routing as shows in the figure 7 where there are a large numbers of red lines and intermediate nodes (black circles surrounded by green). Table 4 shows the importance of a particular airport failure and its impact on topological features, resilience metrics and traffic re-routing costs. For each measure, the bold text represents the highest value of the corresponding metric found in the AAN.

**Figure 7: Traffic re-routing and sensitivity of Sydney airport failure**

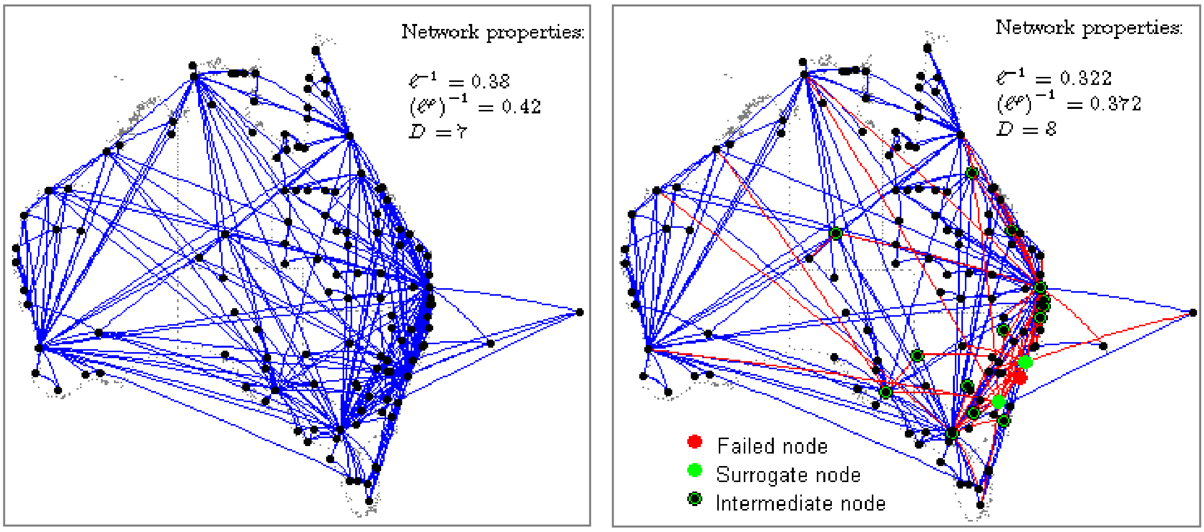



**Table 4: Node importance in terms of resilience measures**

| Airport | Network Features | | | Resilience Measures | | | | | ***Re-route cost per day (in thousands)*** |
|---|---|---|---|---|---|---|---|---|---|
| | $k_i$ | $S_i$ | $B_v(i)$ | $N_c$ | $\Delta L$ % | $\Delta\widetilde{\ell^{-1}}$% | $\Delta\widetilde{D}$% | $\Delta\widetilde{R}$% | |
| Sydney | **99** | **114052** | *0.2940* | *8* | 7.13 | **15.90** | 14.29 | 11.92 | 78.05 |
| Brisbane | *81* | 72199 | **0.3118** | 4 | **17.18** | *14.12* | **100.00** | 6.79 | 29.48 |
| Melbourne | 66 | *85033* | 0.1172 | 3 | 1.33 | 4.77 | 0.00 | 3.05 | **83.03** |
| Perth | 57 | 33345 | 0.1785 | 5 | 6.00 | 10.81 | 14.29 | 9.00 | 57.80 |
| Adelaide | 47 | 35242 | 0.1475 | **9** | 11.54 | 11.77 | 0.00 | *11.93* | 47.99 |
| Cairns | 46 | 18289 | 0.1846 | 6 | 10.02 | 13.28 | 28.57 | **12.62** | 20.35 |
| Darwin | 41 | 9585 | 0.1410 | 7 | *11.86* | 11.40 | 0.00 | 11.89 | 19.44 |
| Townsville | 29 | 10450 | 0.0621 | 2 | 0.05 | 2.68 | 0.00 | 1.53 | 32.05 |
| Gold Coast | 25 | 16513 | 0.0199 | 2 | 1.13 | 2.03 | 0.00 | 1.53 | 3.79 |
| Canberra | 24 | 21681 | 0.0087 | 2 | 1.16 | 2.00 | 0.00 | 1.53 | 15.99 |
| Broome | 21 | 2824 | 0.0116 | 2 | 1.18 | 1.96 | 0.00 | 1.53 | 9.90 |
| Avalon | 20 | 3463 | 0.0196 | 2 | 0.86 | 2.13 | 0.00 | 1.53 | 1.33 |
| Alice Springs | 19 | 3159 | 0.0029 | 2 | 1.14 | 2.00 | 0.00 | 1.53 | 5.40 |
| Mount Isa | 18 | 2530 | 0.0949 | 2 | 3.69 | 3.55 | *42.86* | 1.53 | 1.29 |
| Launceston | 16 | 5535 | 0.0333 | 3 | 2.54 | 3.33 | 0.00 | 3.05 | 3.88 |
| Newcastle | 16 | 7588 | 0.0013 | 2 | 1.32 | 1.76 | 0.00 | 1.53 | 3.82 |
| Karratha | 15 | 3912 | 0.0044 | 2 | 1.27 | 1.82 | 0.00 | 1.53 | 3.40 |
| Mackay | 14 | 6684 | 0.0016 | 2 | 1.29 | 1.78 | 0.00 | 1.53 | 8.59 |
| Rockhampton | 14 | 5759 | 0.0016 | 2 | 1.29 | 1.78 | 0.00 | 1.53 | 1.56 |
| Geraldton | 14 | 1563 | 0.0008 | 2 | 1.54 | 1.52 | 0.00 | 1.53 | 1.13 |
| Port Hedland | 12 | 2424 | 0.0018 | 2 | 1.33 | 1.73 | 0.00 | 1.53 | 1.66 |
| Learmonth | 12 | 629 | 0.0003 | 2 | 1.55 | 1.50 | 0.00 | 1.53 | 0.39 |
| Kalgoorlie-Boulder | 10 | 1447 | 0.0018 | 2 | 1.38 | 1.70 | 0.00 | 1.53 | 2.54 |
| Hobart | 10 | 7100 | 0.0000 | 2 | 1.33 | 1.72 | 0.00 | 1.53 | 1.70 |

In Table 4, we can see that Sydney is the most connected airport in the network and also handles the highest amount of traffic as it has the highest strength value. If it is removed from the network, sensitivities measure by of the characteristics path length and network diameter will increase to $\Delta L = 7.13\%$ and $\Delta\widetilde{D}\,\%$=14.29%, respectively. Failure of Sydney airport will also decrease the efficiency of the network ($\Delta\widetilde{\ell^{-1}} = 15.90\%$) and it will cost 78 thousand units of re-routing cost per day, the second highest after Melbourne.

As Brisbane is the second highest in terms of connectivity and most central node in the network (highest betweenness value), it serves as an important bridge in the network and, as a result, if it breaks down, will increase the network's diameter to twice $(\Delta\widetilde{D} = 100\%)$ of its original size; for example, it requires 14 hops to travel from Taree to Birdsville or vice versa but only 7 if Brisbane is connected. Therefore, removing Brisbane from the network will significantly slow down the physical movements of passengers and goods across the AAN. From its high betweenness value and the effect of the increase in its diameter, it is clear that, if we want to protect, or break down the flow of any material or disease, Brisbane would be the first place to stop it.

Although Melbourne is found to be one of the least sensitive and has no impact in terms of changing the diameter of the network, it would have the highest re-routing cost of 83.03k per day. The reason for this is its geographical location in the network and its lack of any close airports that could serve as surrogates. If we look at the reachability measures, only the top 7 connected airports have a significant impact on it, with Cairns found to be the most sensitive.

Canberra has high degree and betweenness values but a significant re-routing cost because of its geographical location in the network and airports in close proximity that could act as surrogate airports for heavy aircraft. As a consequence, all heavy aircraft have to be re-routed to Sydney which significantly increases the re-routing cost.

**Figure 8: Effect of different node failures on the traffic re-routing cost**

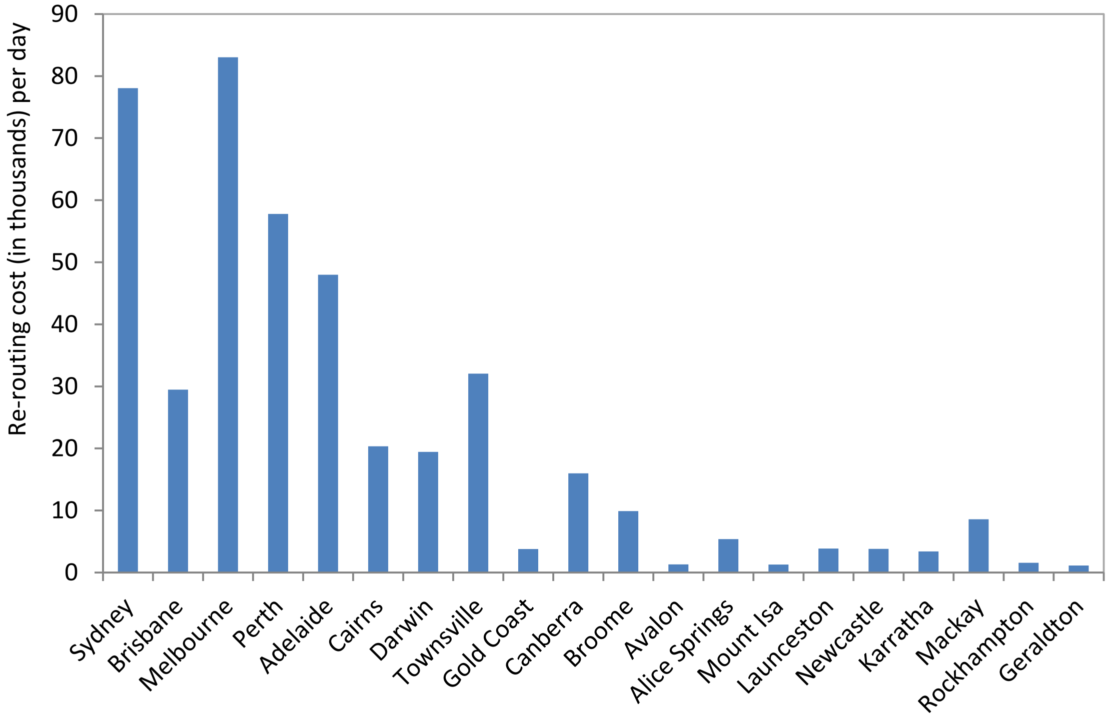



Figure 8 shows the re-routing costs for the top 20 connected airports in which we can see that the most connected are not always the most costly, with the most important nodes in terms of cost being Melbourne and Sydney because of their high strengths and geographical locations. However, although Brisbane is also a highly connected airport and, in fact, the second highest, it has a low re-routing cost because it has a very close surrogate airport, i.e., Gold Coast, which is only approximately 100km away and capable of handling all type of aircraft (light, medium and heavy). From this analysis, we find that the re-routing cost is highly correlated with the geographical location of an airport and the availability and capability of any nearby surrogate airport.

Table 5 lists the correlations among the network features and traffic re-routing costs of the AAN in which significant levels are observed: nodes that have large degrees also have typically large strength and betweenness values; the re-routing cost is more highly correlated with strength than betweenness; the clustering coefficient $(C_i, C_i^w)$ is negatively correlated with all other network features as well as the re-routing cost because the cost of adding a route/link does not depend on the geographical constraint in the case of an airport network. It is relatively easier to add an air connection to an existing network than to add a data or electrical cable for comparable geographical distances.

**Table 5: Similarities of features and re-routing costs of AAN**

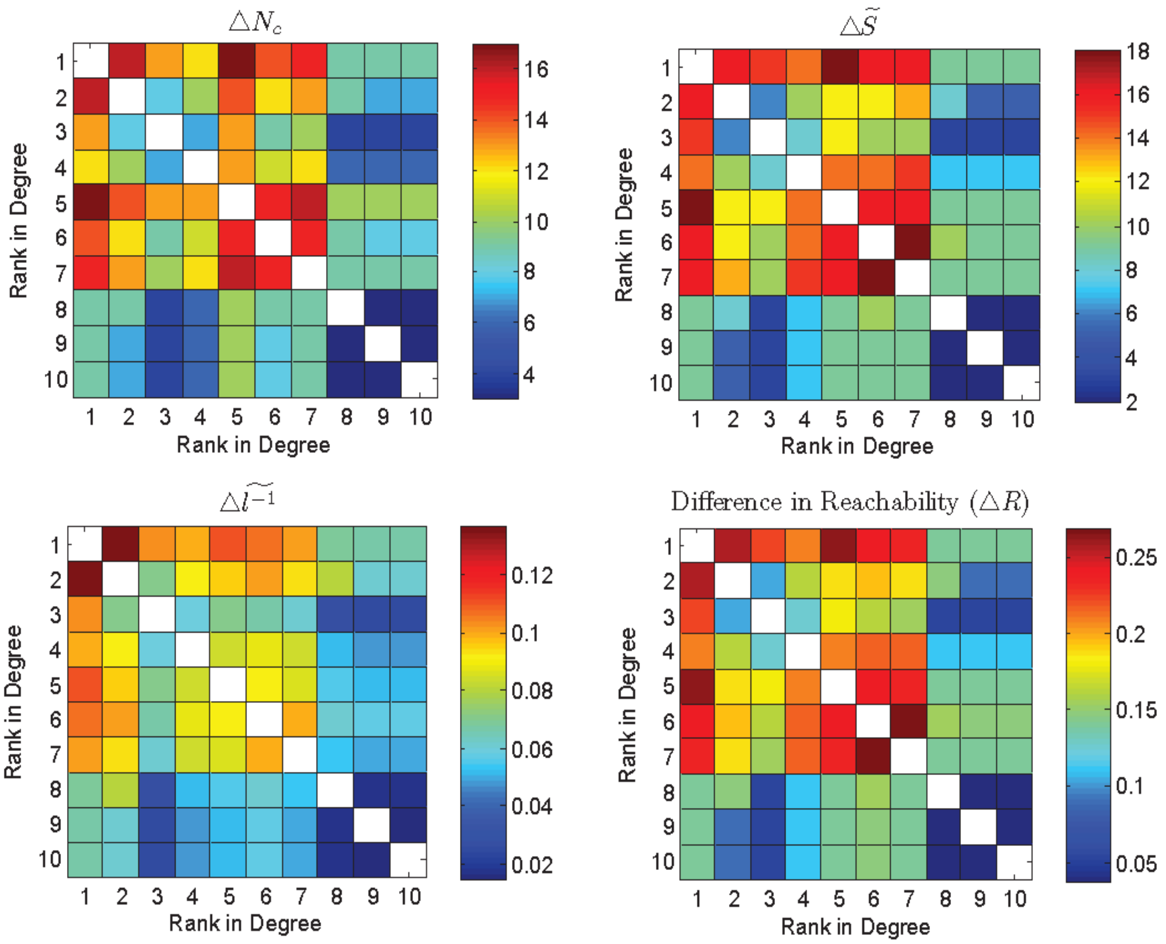
| **Correlation coefficient** | $\boldsymbol{k_i}$ | $\boldsymbol{C_i}$ | $\boldsymbol{C_i^w}$ | $\boldsymbol{S_i}$ | $\boldsymbol{B_v(i)}$ | $\boldsymbol{Reroute\ Cost}$ |
|---|---|---|---|---|---|---|
| Degree, $k_i$ | 1 | -0.1094 | 0.0896 | 0.9221 | 0.9276 | 0.8956 |
| Clustering coefficient, $C_i$ | | 1 | 0.5928 | -0.1425 | -0.1923 | -0.1461 |
| Clustering coefficient weighted, $C_i^w$ | | | 1 | 0.0093 | -0.102 | 0.0055 |
| Strength, $S_i$ | | | | 1 | 0.8395 | 0.9076 |
| Node betweenness, $B_v(i)$ | | | | | 1 | 0.7907 |
| $Reroute\ Cost$ | | | | | | 1 |

Although the high betweenness nodes are the most central in terms of information sharing or load carrying, it is far easier to obtain a surrogate node which ensures that costs remain low. From the analysis presented in this section, it can be summarised that high central nodes are not always the most critical in terms of vulnerability and that resilience cannot be measured by a single metric. However, it remains unclear as to how sensitive these measures are for assessing which combinations of failed nodes would have the most significant impact on the AAN's vulnerability.

To gain an insight into this issue, we couple two node failures according to their degrees and compare their resilience measures with those of the original AAN and, to visualise the effects, we plot a coloured map for each metric. In this analysis, the original AAN network is considered a baseline network with which the modified (after two high-degree node failures) networks are compared, with the effects highlighted in the coloured maps. To capture the changes after this pair of node failures, we calculate each metric's difference from its corresponding value in the original AAN. Figure 9 shows the effect of the resilience measures after the simultaneous failures of a pair of high-degree nodes in which the axis represents the rank of a node according to its degree, where rank 1 indicates the highest degree. Since these maps show the effects of a pair of node failures, as the right diagonal value represents only one node failure, its value is omitted and all its elements left white.

The top left coloured map in Figure 9 shows changes in the numbers of components ($\Delta Nc = Nc - number\ of\ component\ in\ original\ AAN$) in which it can be seen that failure of the pair with ranks 1 and 5 (Sydney and Adelaide) is the most affected. The top right, bottom left and bottom right maps show changes in size of the giant component ($\Delta\tilde{S}$) from that of the existing AAN), average inverse geodesic distance ($\Delta\ell^{-1}$) and reachability difference, ($\Delta R$), respectively.

**Figure 9: Coloured maps of pair of high-degree node failure**



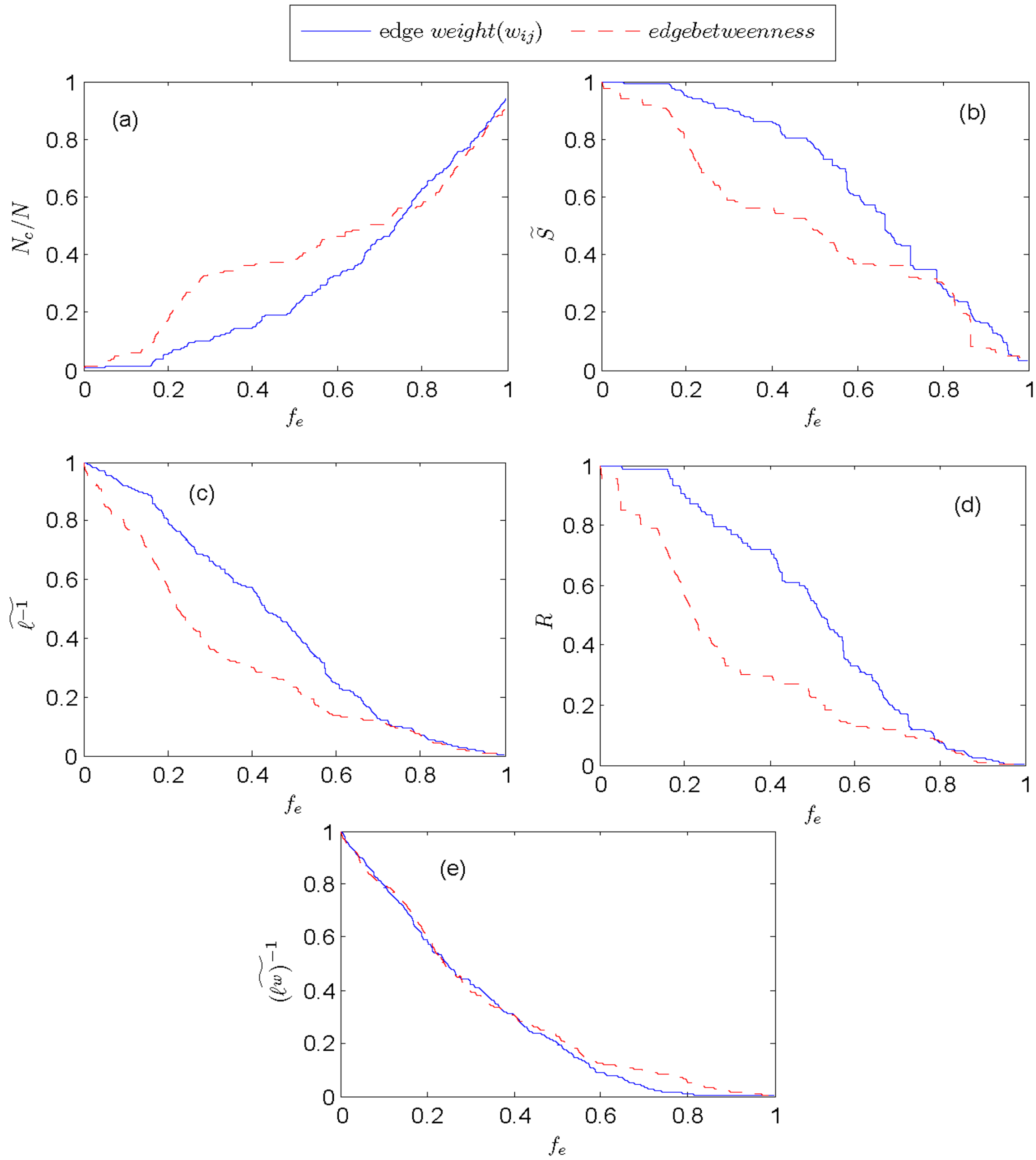

From these maps, it is easily noticeable that the Sydney-Adelaide pair is the most affected according to the $\Delta Nc$, $\Delta\tilde{S}$ and $\Delta R$ measures, with only $\Delta\ell^{-1}$ having a discrepancy for which the Sydney-Brisbane pair is the most affected while, in the $\Delta\tilde{S}$ and $\Delta$R measures, the Cairns-Darwin pair has the second highest effect. However, for each of the pairs, row or column 3 has less impact which means that, if Melbourne is involved in the failure pair, on average, it will have a lower impact than the others. Also, it can be observed that the bottom part of each map always has less impact and it is clear that only the top 7 connected airports have more significant impacts on the resilience measure than the others.

## 5.3. Tolerance to Edge Failure

Apart from node failures (airport shutdowns), we also investigate the vulnerability of the AAN subjected to various types of edge failure scenarios. Figure 10 shows the results for the resilience or vulnerability to edge failure according to a descending order of weights (amount of traffic) and edge betweenness (edge bottleneck). When the edges are removed, the total number of nodes ($N$) does not change which makes $\ell^{-1}$ a monotonically decreasing function with the fraction of removed edges ($f_e$).

In Figure 10(a) and 10(b), it can be seen that the network remains connected until 20% of its highly weighted links have been removed although this is not true in the case of the removal of bottleneck edges. However, the number of sub-graphs increases monotonically with increasing numbers of edge failures.

**Figure 10: Resilience measures of AAN against edge failure (links breakdown)**




It can also be inferred from Figure 10 (a) and 10(b) that the highly weighted links are not the most critical links in the network as, for all the measures (topological sensitivity and reachability), removing bottleneck edges (edge betweenness) has much more effect than removing highly weighted links (edge weight) which suggests that edge betweenness is a more suitable quantity than weight for measuring the importance of an edge (link). Of all four metrics, the greatest differences between the two edge-failure strategies are observed in the range from $f_e = 0.25$ to $f_e = 0.5$ but, after 80% of edges are removed, they are identical. After 25% ($f_e = 0.25$) of bottleneck edges break down, the reachability ($R$) drops sharply to almost 30% of its original network value.

In the analysis summary presented in this section, it is notable that, in the case of node failures (both degree- and betweenness-based), all the resilience metrics decrease sharply in contrast to their behaviour for an edge failure. Due to the hub and spoke nature of the AAN, a highly connected node failure is the most destructive of all the resilience measures.

However, because of the heterogeneity of the weights associated with its links, the network is fairly robust to edge failures but vulnerable to high-degree node failures.

## 6. Conclusion

In this paper, the infrastructure of the AAN was analysed from a complex network theory point of view. The AAN model was constructed by associating a node with each airport and creating a link to each directly connecting passenger flight operating between different airports using air traffic data from the year 2011. The results indicated that the AAN is a small-world structure in which the numbers of non-stop connections from, and shortest paths going through, a given airport have scale-free distributions that indicate the presence of high-degree nodes (called hubs), particularly the three largest airports (Sydney, Brisbane and Melbourne). The traffic (number of flights) of the AAN was found to accumulate on an interconnected group of high degree nodes.

These analyses provided valuable information about the characteristics of the AAN and the level of vulnerability to which it can be exposed through random, most central nodes and link failures. The study of the responses of the AAN subjected to different node and edge failure scenarios showed that it is comparatively robust in terms of edge or air-route shutdowns but very sensitive (less robust) to central node failures, particularly of the high-degree and high-betweenness nodes.

We defined the importance of a node closing or shutting down in terms of the traffic re-routing cost and found the cost of such a node failure depends on its physical location in the network and is highly correlated with its strength. Airports those are closely surrounded by other airports that can handle the same or even better classes of aircraft usually have low sensitivity in terms of traffic re-routing.

The analysis also revealed that the most central nodes are not always the most critical. Most importantly, the robustness of an airport network cannot be expressed quantitatively by a single measure as it is a complex combination of several topological and operational metrics. Although the focus of this research was on the AAN, its results and insights are applicable to other airport or transportation networks with similar network features. This paper presented various network parameters that could potentially be used as measures of the performance of, and risks to, an airport network. However, as the further integration of social and economic rules governing airline operations remains an important challenge for achieving a comprehensive understanding of the socio-economic dynamics associated with the AAN, these properties will be examined in future together with the role of the AAN in the larger context of the WAN.

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