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Park-and-Ride network design in a bi-modal transport network optimising network reliability

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Abstract

Modern cities suffer immensely from traffic congestion caused mainly by the proliferation of private vehicles on the road network. Park-and-Ride (P&R), an already established critical infrastructure system, encourages commuters to drive from their trip origin to train stations or bus terminals, where they park and ride public transport (bus, train or tram) to their destination with the objective of mitigating traffic congestion. Traffic congestion adversely affects the roadway capacity thus making the traffic condition on the network uncertain. Therefore, the need for reliable transport system is an increasing demand for transport network users. Reliability of network is one of the key indicators for the evaluation of transport network performance. Optimisation of network reliability for P&R network has not been studied yet. Consequently, the P&R network design in a bi-modal transportation system considering the network reliability is a novel area in transport research. Commuters manifest different types of behaviour when tackling unstable traffic condition in transportation network. This behaviour can be known as risk-aversion, risk-neutral and risk-seeking. Travel time disutility associated with the aforementioned behaviours is taken as reliability measurement for the evaluation of network performance. A Bi-level programming model is developed to determine the optimal P&R scheme in a bi-modal transport network with the objective of optimising network reliability. Optimal P&R scheme includes optimal parking location and size which are determined considering real life commuters behaviour. The objective of the upper level is to minimize the total expected travel disutility in the whole network. The lower level is the commuters' route choice problem and travellers are assumed to choose the path/route with minimal expected travel disutility. This disutility-related multiclass user equilibrium is formulated as nonlinear complimentary problem which is a non-additive path cost problem. A Genetic Algorithm (GA) is implemented to solve the bi-level model and verified by a numerical example considering a P&R network.

1. Introduction

Due to the limited capacity of transportation infrastructure, traffic congestion and associated environmental pollution as a result of gas emissions deteriorating the quality of life and safety in many modern metropolis. Consequently, tackling such congestion issues has been a dominant theme of concern in transport research. Most countries around the world are now turning to traffic management systems to cope with the congestion (Manns, 2010). Transit has been promoted in many cities around the world to address people's travel needs (Qin et al., 2013). It has been considered an effective way to mitigate the growing traffic congestion by encouraging public transport use upon implementation of the congestion pricing regimes to increase network users' travel cost of using private cars in CBD area (Liu and Meng, 2014). Hence, public transport facilities have been viewed by the transport engineers, transit operators or urban planners as a solution to the traffic congestion related problems. The consideration of various forms of public transportation, increase in the coverage of public transport systems, higher passenger ridership and more affordable fare structure are common measures to increase the public transportation usage (Rosli et al., 2012). Among the different strategies, Park-and-Ride (P&R) has been used as a means of travel demand management throughout many western countries since the 1930s (Noel, 1988). P&R is defined as an operation in which commuters, travelling by private vehicle either as drivers or as passengers, transfer at a parking site to higher-occupancy public vehicles i.e. light or heavy trains, buses and trams to complete their journeys. Thus P&R is regarded as an appropriate way to increase the public transport mode share.

Travel time is one of the major aspects of transportation system. Travel time varies with time and such variability in travel time is due to unsteady and various characteristics of key conditions which are the direct source of traffic congestion (Hojati et al., 2009). There are two types of congestion in the network known as recurrent and non-recurrent congestions. Recurrent congestions are due to everyday over exceeding travel demand during peak traffic flow, while non-recurrent congestions are due to unpredictable and unsteady situations (Hojati et al., 2009).This traffic congestions adversely affect the roadway capacity thus make the traffic condition on the network unreliable. Establishing network reliability has become a significant concern for the transport authorities in the design of transportation infrastructure and network. Therefore, as a measure of network reliability, travel time reliability is considered as one of the key assessment indicators for evaluating transportation network performance.

Different kinds of measurement analysis and definition of reliability of network are present in literature. Lam et al has utilized traffic flow simulator to investigate the transportation network reliability in which the variation of perceived travel time error and the fluctuations of origin-destination (OD) demand are explicitly considered (Lam and Xu, 1999). Another measurement of travel time reliability is carried out using travel time variance derived from the path flow estimator (Bell and Iida, 1997). Considering utility of users as the measure for network performance reliability has been utilized in one previous work by Bell (Bell, 1999). Yin and Ieda uses different risk-taking behaviour of commuters while they are challenged with uncertain traffic conditions on the network as a result of non-recurrent congestions. The different decision-making behaviours of commuters are examined using disutility approach and commuters are assumed to choose path with the minimal expected disutility between origin and destinations (Yin and Ieda, 2001). Travel disutility as defined in their work refers to the dislike associated with the level of the travel time variations perceived by commuters is considered as a measurement index for network reliability in network design.

Modelling of P&R services is found in some literature. Bo Du et.al. have considered continuum modeling of P&R services for a linear travel corridor (Du and Wang, 2014). In their work, equilibrium travel pattern is formulated as linear complimentary system. Another work is found on linear complimentary system modeling approach for P&R services in the work by Wang et al (Wang and Du, 2013). But the effect of location and parking size of particular P&R scheme on the network reliability of transportation network as a whole have not been looked at. Optimisation of network reliability for P&R network has not been studied yet. The paper adapts the travel disutility measurement method of Yin and Ieda (Yin and Ieda, 2001) to optimize the network reliability of a bi-modal transport network consisting of auto and P&R sub-network. This paper outlines a methodology to find the optimal P&R pattern. The optimal P&R scheme determines optimal location and capacity of the P&R facility on a bi-modal transport network which optimises the network reliability. Thus it commences with the optimization method which is formulated as a bi-level programming problem with the explanation of each level given separately in Section 2. This is followed by a Genetic Algorithm based solution algorithm. Finally a conclusion, including a discussion of future direction of research in this field, is offered.

2. Bi-Level Problem Statement

P&R network design problem is a generalized Network Design Problem (NDP). NDP describes a hierarchical system that is composed of two levels of decision makers (Migdalas, 1995). In this paper, there are two levels of stakeholders who ascertain the performance of a P&R scheme. The upper level is the transport authority who would propose a network where a particular P&R scheme is provided. Given this P&R scheme, at the lower level, transport network users would choose a strategy to maximize their own benefit under the prevailing conditions. This bi-level problem is modelled as a Stackelberg duopoly game problem (Yang and H. Bell, 1998). In equilibrium conditions, the optimal P&R scheme is chosen.

The upper level is concerned to improve the efficiency of the whole transportation system within a limited budget and other issues thus their perspective is considered in the upper level. In this paper, the objective of the upper level is to minimize the total travel disutility and hence maximize reliability in the network. The authority takes into account the total travel disutility of auto and P&R users. Several practical constraints for the P&R scheme are formulated as the upper level constraints. Following subsection defines a comprehensive objective function and associated constraints. The output of the upper level is the set of two decision variables which define the location of P&R facility and corresponding size of the site. Therefore, in this paper, these two types of decision variables that are involved into P&R network design problem, encompass the location (x ∈ X) for the P&R facility, which is a binary (0-1) variable and the capacity of the parking size of that facility (y∈ Y), which is an integer variable ranging in$\underline{Y} \leq Y \leq \overline{Y}$.

Commuters at the lower level are considered to exhibit different risk-taking behaviours when challenged with unstable travel conditions in the transportation network. Different behaviours of network users namely, risk aversion, risk neutral and risk seeking can be formulated by the expected disutility approach (Yin and Ieda, 2001). Commuters are assumed to choose route with minimal expected travel disutility. Therefore, at the lower level, disutility-related multi-class user equilibrium tool is used to solve the traffic assignment problem. Essentially, the P&R scheme determined at the upper level is evaluated based on the result given by the user equilibrium model achieved in the lower level.

A bi-modal transportation network is denoted by G= (N, A), where N is the set of nodes and A is the set of links connecting nodes. The Bi-modal transportation network G consists of auto sub network and P&R sub network. The auto and rail links are connected with corresponding P&R site to make up the P&R links the network. The detail formulation of the upper level and lower level are described in the subsequent sections.

2.1 Upper Level problem formulation

Optimal P&R scheme is determined using the reliability measurement approach at the upper level. The reliability of the transport network is defined as the total disutility of commuters in the network (Yin and Ieda, 2001). The aim of the transport authority is to optimise the network reliability in the whole transportation system. Therefore, the concept of measuring the reliability in the whole transport network in terms of the total travel disutility has been formulated as upper level objective function. Thus in this paper, the total travel disutility in the bi-modal transportation system is the sum of the travel disutility of auto users and P&R users. The objective is to find out the P&R scheme whose corresponding total travel disutility is minimum. The mathematical formulation of the upper level is proposed as follows.

$$\min Z=\sum_{i\in I} \sum_{od} \pi^{od}_{a,p\&r,,i} (X, Y)\ q^{od}_{i} \quad (1)$$

subject to:

$$\underline{Y} \leq Y \leq \overline{Y} \quad (2)$$

$$\sum_{j} cy < \bar{C} \tag{3}$$

$$R \geq R_{th} \tag{4}$$

$$Y<MX \tag{5}$$

where:

a represents auto routes (paths)

p&r represents park and ride routes (paths)

OD : Set of O-D pairs for the whole network

P^{od} : set of all paths between OD pair

I : 1, 2 and 3 corresponding three types of commuters as risk-averse, risk-neutral and risk-prone respectively

q_i^{od} : Travel demand between OD pair (o, d)

$\pi_{a,p\&r,i}^{od}$: Minimum expected path disutility for OD pair *od* and *i*th class of commuters, in other words, $\pi_{a,p\&r,i}^{od}$ = min [$\eta_{a,i}^{od}$, $\eta_{p\&r,i}^{od}$] for all paths of the OD pair

$\eta_{a,i}^{od}$: Expected path disutility for auto paths of OD pair *od* =E [$U_i(t_a^{od})$] =$\int U_i(t_a^{od})\ dt_a^{od}$; where, $t_a^{od} \sim N(\mu_a^{od}, \sigma_a^{od^2})$

$\eta_{p\&r,i}^{od}$: Expected path disutility for P&R paths of OD pair *od*

=E [$U_i(t_{p\&r}^{od})$] =$\int U_i(t_{p\&r}^{od})\ dt_{p\&r}^{od}$; where, $t_{p\&r}^{od} = N(\mu_{p\&r}^{od}, \sigma_{p\&r}^{od^2})$

E(.) represents expected value

$N(\mu, \sigma^2)$ represents a normal distribution of path travel time with mean μ and variance σ^2

$U_i(.)$ is disutility function, which is a usually a upward convex function of path travel time for risk averse commuters, a downward concave function of path travel time for risk seeker commuters and a positive-sloped linear function for risk neutral commuters.

The objective function minimises the total network travel disutility by considering both auto and P&R users. Constraint (2) is size limitation for the parking sites. Constraint (3) demonstrates the budget constraint as the objective function in Equation (1) is formulated from transport authority's perspective. Constraint (4) is the reliability constraint of the network, where, reliability is defined as: $R=\frac{\sum_i \sum_{od} \pi_{a,p\&r,i0}^{od} q_i^{od}}{\sum_i \sum_{od} \pi_{a,p\&r,i}^{od} q_i^{od}}$; $\pi_{a,p\&r,i0}^{od}$ = the minimal expected disutility between OD pair *od* for class *i* commuters by considering all link travel times are deterministic. The designed network must have a reliability greater than or equal to a pre-defined threshold R_{th}. It should be noted that *R* has a maximum normalised value of 1.0. Constraint (5) determines the relationship between x_i and y_i, that y_i is determined by x_i. When $x_i = 0$, y_i should also be 0. When $x_i \neq 0$, y_i can be a random value between $\underline{Y}$ and $\overline{Y}$. It should be noted that, in this paper, the link travel time, as well as path travel time are assumed to follow a normal distribution (Yin and Ieda, 2001).

2.2 Lower Level formulation

Once the P&R scheme is determined, it is now time to find out its performance. The users decide on the utilization of the available P&R scheme to them. Therefore, the lower level model evaluates the performance of the P&R scheme.

User equilibrium assignment is such a tool that can be used to predict the traffic flows on the network. This paper considers the commuters with different kinds of risk-taking behaviours and thus the disutility approach has been utilized to examine the behaviours for multi-class users. The disutility-related multiclass user equilibrium is formulated as nonlinear complimentary problem (NCP) which has been previously used by Yin and Ieda (Yin and Ieda, 2001).

According to the conditions of disutility related multi-class user equilibrium, it is assumed that every commuters will try to minimize their own travel disutility while travelling from origin to destination (Yin and Ieda, 2001). As a result, the network equilibrium will be reached when no commuters can change his or her expected travel disutility by unilaterally changing routes.

The disutility related multi-class UE is conducted on the bi-modal network which is a combination of auto and P&R. The auto commuters transfer to rail mode through the corresponding parking facility. The time cost of car parking increase with the increase of P&R users. The travel time taken by rail link is set to be a constant value.

In this paper, the model for the disutility related multi-class UE problem based on Yin and Ieda (Yin and Ieda, 2001) is adapted. Therefore, the mathematical formulation of the lower level problem which is formulated as equivalent NCP (Lo and Chen, 2000) is as follows:

$$\min, G(X)$$

subject to:

$$X \in [X \geq 0 \quad F(X) \geq 0] \tag{6}$$

The corresponding UE traffic assignment conditions are stated as follows:

$$\eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od} \geq 0 \qquad \forall od \in OD, \forall p \in P^{od}, \forall i \in I \tag{7}$$

$$(\eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od}) f_{p,i}^{od} = 0 \quad \forall od \in OD, \forall p \in P^{od}, \forall i \in I \tag{8}$$

$$\sum_{p \in P^{od}} f_{p,i}^{od} - q_i^{od} = 0 \quad \forall od \in OD, \forall i \in I \tag{9}$$

$$f \geq 0, \pi \geq 0 \tag{10}$$

Conditions (7) and (8) define the complementary slackness condition: $f_{p,i}^{od}$ =0, $(\eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od})$>0 and $(\eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od})$=0, $f_{p,i}^{od} \geq 0$; that is, when the expected disutility on path p is larger than the minimal disutility, the flow on that path is zero. When the expected disutility on path p is equal to the minimal disutility, the flow on that path is equal to or greater than zero. These complementary slackness conditions are equivalent to the user equilibrium. Conditions (9) and (10) are the flow conservation, non-negative constraints respectively.

The equivalent NCP formulation of the UE conditions are (Lo and Chen, 2000):

$$X \geq 0,\ F(X) \geq 0,\ X^T F(X) = 0 \tag{11}$$

with

$$X = \binom{f}{\pi} \text{ and } F(X) = \binom{F^f(X)}{F^\pi(X)} \tag{12}$$

where, $F^f(X)$is the column vector of$(\eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od}, \forall od \in OD, \forall p \in P^{od}, \forall i \in I)$, and $F^\pi(X)$is the column vector of $(\sum_{p \in P^{od}} f_{p,i}^{od} - q_i^{od}, \forall od \in OD, \forall i \in I)$. Aashtiani (Aashtiani, 1979) established the NCP in Equation (6) and (7) and proved the existence and uniqueness of the solution of this NCP in π.

The definitions of terms are:

P^{od}= The set of all paths between OD pair

$f_{p,i}^{od}$=The flow of class i commuters on path p between O-D pair *od*

$\eta_{p,i}^{od}$ =expected disutility on path/route *p* between OD pair *od* for class *i* commuters

f = The vector of path flow, $f_{p,i}^{od}$with dimension n_1=$\sum_{od}|P^{od}|$x 3 that equals three times total number of paths in network

π = Vector of minimal expected disutility π_i^{od} with dimension n_2=$|OD|$x3 that equals three times total number of O-D pairs

G(X) = The gap function to reformulate the NCP, based on Yin and Ieda's work (Yin and Ieda, 2001)

where,G(X)=$\sum_i \sum_{od} \sum_p \sum 1/2\,[\sqrt{(f_{p,i}^{od})^2 + \left(\eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od}\right)^2}\; - (f_{p,i}^{od} + \eta_{p,i}^{od} - \pi_{a,p\&r,i}^{od})\,]^2 + \sum_i \sum_{od} 1/2[\sqrt{(\pi_{a,p\&r,i}^{od})^2 + \left(\sum_{p\in P^{od}} f_{p,i}^{od} - q_i^{od}\right)^2}\; - (\pi_{a,p\&r,i}^{od} + \sum_{p\in P^{od}} f_{p,i}^{od} - q_i^{od})]^2$ (13)

3. Solution Algorithm

The bi-level programming problem is a NP-hard problem and is relatively difficult to solve even for a medium size problem (Ben-Ayed et al., 1988). In this paper, Genetic Algorithm (GA) is chosen to solve the proposed model. The GA is the most well-known heuristics for solving optimization problems (Liu and Meng, 2013). GA begins with a feasible set of answers called population by mimicking the natural selection process. Each individual answer in the population is called a chromosome and assigned a survival probability based on the value of the objective function (Mesbah and Sarvi, 2009). GA uses cross over and mutation operators to breed the next generation which replaces the predecessor generation (Mesbah and Sarvi, 2009). The algorithm is repeated with the new generation until a stop condition is reached. Common stop criteria include: fixed number of generations is reached, predetermined computation time is reached, the difference of the optimal solution between the two adjacent generations cannot be further reduced, the difference of the worst solution between the two adjacent generations cannot be further reduced and the difference between the optimal solution and the worst solution cannot be further reduced (Chen et al., 2014). Several studies have utilized GA to network design (Fan and Machemehl, 2006);(Guihaire and Hao, 2008).

In this paper, GA is used to find the optimal P&R scheme. To adapt the GA into the problem of this research study, chromosomes of the GA are designed as: all the railway stations in the network are successively numbered and each gene in one chromosome represents the decision of locating the P&R facility which is a binary variable (0-1) and the corresponding size of parking lot which is an integer value between $\underline{Y}$ and $\overline{Y}$. The chromosome, i.e. each P&R scheme, contains a feasible combination of P&R site location and corresponding size. The algorithm starts with an initial generation where all genes are randomly generated to produce an initial feasible population. The detailed steps of the algorithm are given below:

- Step 1: Initial population: let the population size be $P_{size.}$ The maximum number of generations is set to be $i_{max.}$ Set the iteration counter, i=1. Initial population of the chromosomes consisting of location decision X_i and corresponding parking site capacity Y_i

is generated randomly. The crossover and mutation rate are set as P_{xover} and $P_{mutaiton}$ respectively.

- Step 2: Evaluation: once initial population produced, now solve the bi-level problem for each newly generated chromosome. The objective function for all the chromosomes is determined using the value of the minimal expected disutility by solving the lower level traffic assignment problem. To ensure the feasibility, each chromosome not subject to the budget and reliability constraints is multiplied by 100 to make them eliminated in the next generation.

- Step 3: Selection: From the existing chromosomes, select the one with lowest travel disutility i.e. the lower the value of objective function of the chromosomes the higher is the probability to be selected. Also the best chromosome of the previous population is kept in the next generation.

- Step 4: Crossover: Parents are chosen randomly from the chromosomes in the current generation and pairing are conducted between each parents to yield new chromosomes. Only the parking site capacity is to be cross overed. If the parking size falls below the minimum range, $\underline{Y}$ after crossover, then size is to be reset as $\underline{Y}$.

- Step 5: Mutation: Genes from all the chromosomes in the current generations are chosen randomly and the value of chosen genes are modified by a pseudo random number. The mutation contains two stages: mutation of location decision and mutation of capacity. First the decision of location is modified by a 0-1 variable. If the previous decision is 1 and not modified, then modify the capacity by a random integer number between $\underline{Y}$ and $\overline{Y}$. Thus some new chromosomes are generated.

- Sep 6: Termination: Terminate the algorithm if $i > i_{max}$ or if the result does not improve after a considerable number of iterations, otherwise set, i=i+1 and repeat with step 2.

4. Numerical Example

To numerically validate the proposed model, it is applied to a small sized example network. It should be noted that solving particularly the lower level NCP traffic assignment problem becomes intractable as network size increases (Lo and Chen, 2000). The presented network example is built on the basis of a road sub-road network and a P&R sub-network. The network consists of six nodes and ten links as indicated in Figure 1. Among the ten links, seven links are auto links and the remaining three links are rail links used as part of the P&R routes. Two O-D pairs are assumed as 1→3 and 2→4. The origins denote the suburbs while the destination nodes are the city centres. Hence the roads become increasingly narrow and thus congested. The travel demands are assumed to be fixed at 300 for both OD pairs i.e. $D_{1\text{-}3}=D_{2\text{-}4}=300$. The link attributes for auto links and for P&R links are presented in Table 1 and Table 2 respectively. The Bureau of Public Roads link travel time function is used as shown in Equation (14) and equation (15).

$$\mu_{la}(v_{la}) = t_{la}^{0}[1+0.15.(\frac{v_{la}}{C_{la}})^{4}] \tag{14}$$

$$\mu_{lp\&r}(v_{lp\&r}) = t_{lp}^{0}[1+0.15.(\frac{v_{lp}}{y_{i}})^{4}] + \mu_{lr} \tag{15}$$

where, t_{la}^{0} is the auto link free flow travel time, v_{la} is the auto link flow and C_{la} is the auto link capacity. t_{lp}^{0} is the free flow time required for parking a car while transferring from road link to rail link for a P&R route, v_{lp} is the rail link flow i.e. equals to P&R flow and y_i is the parking size, μ_{lr} is the subsequent rail link travel time.

Figure 1: Example bi-modal traffic network

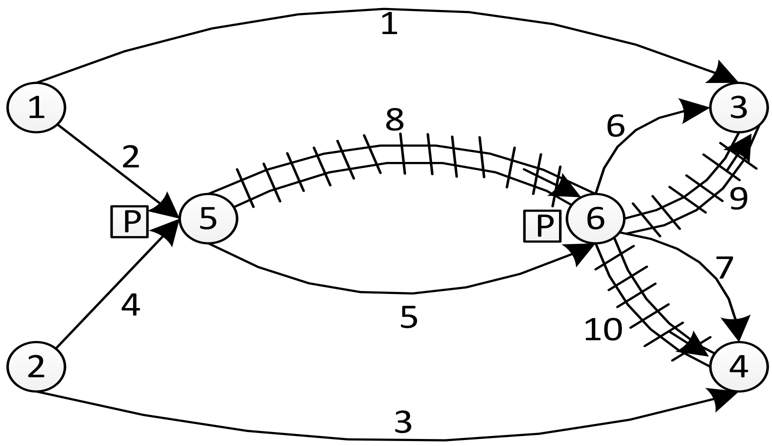


Table 1: Input data for auto links of the example network

Parameter	Auto link						
	1	2	3	4	5	6	7
t_a^0	12.5	2	14.5	3	6	3	4
$\bar{c_a}$	200	200	200	200	400	40	50

Table 2: Input data for P&R links of the example network

Parameter	P&R link		
	8	9	10
$t_{p\&r}^0$	0.05	0.05	0.05
$\mu_{p\&r}$	7	2.9	3.9

Each OD pair has three paths (two paths for auto mode and one path for P&R mode) as described in Table 3. Of all the six nodes, four nodes are considered as railway stations. Among the four railway stations, two nodes (node 5 and node 6) are stations to be considered as P&R facility. Only one site will be selected for P&R facility due to the budget constraint. Therefore, in Table 3, path 3 for OD pair 1-3 is designated as 2-8-9 when P&R facility is located at node 5 or 2-5-9 for P&R facility located at node 6. This will be the case for OD pair 1-4 as well where path 3 will be either 4-8-10 or 4-5-10. It is noted that, in this example, the commuters are assumed to be risk averse only. The expected path travel disutility for risk-averse commuters is represented as below:

$$\eta_p^{od}=\sum_{l\in L^{od}} \delta_{l,p}^{od}\,\beta_l^2\mu_l^2 + \sum_{l\in L^{od}}(\delta_{l,p}^{od}\,\mu_l)^2 + \sum_{l\in L^{od}} \delta_{l,p}^{od}\,\mu_l \quad (16)$$

where,

L^{od}: The set of all links between OD pairs *od* in the network;

β_l: A link related constant which represents the link travel time variation

μ_l : The expected link travel time and standard deviation of the link travel time is $\sigma_l=\beta_l\mu_l$;

μ_p^{od} : Expected path travel time $=\sum_l \delta_{l,p}^{od}\ \mu_l$ and $\delta_{l,p}^{od}$ =1 if link l is a part of path p connecting OD pair *od* and $\delta_{l,p}^{od}$=0 otherwise.

The travel time variations are considered to be on link 5 (β_5) i.e. only link 5 can be stochastic and all other remaining links are considered to be deterministic.

Table 3: OD pairs and corresponding paths for both auto and P&R mode

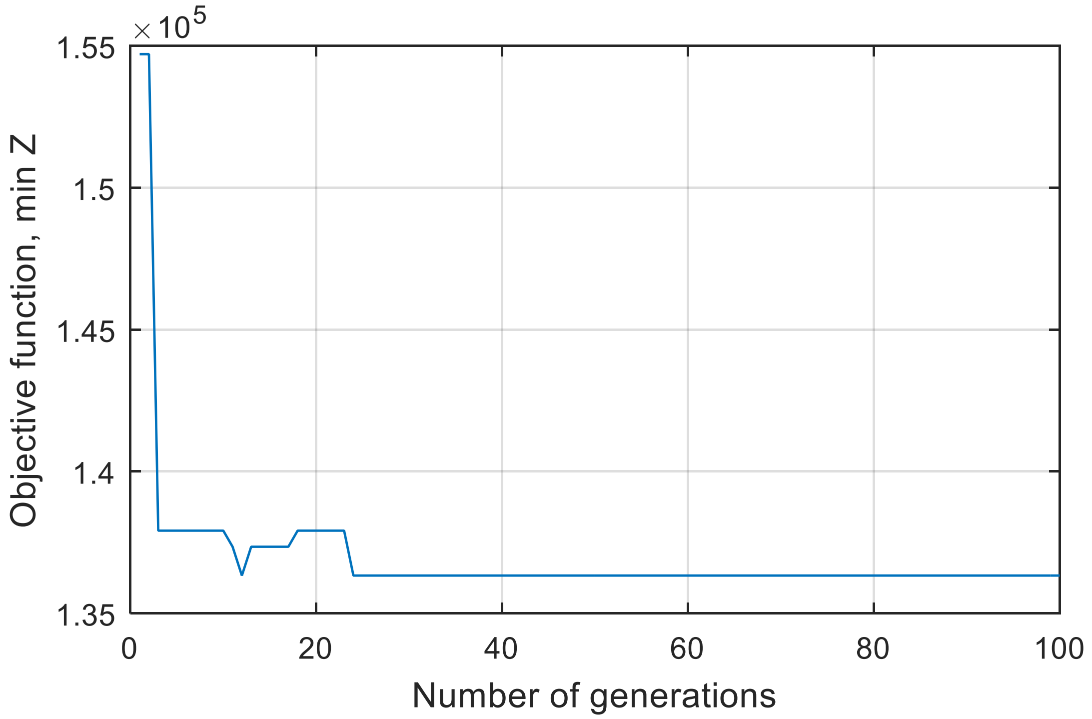
	OD Pair 1: Node 1 to Node 3	OD Pair 2: Node 2 to Node4
Path 1 (Road)	Link 1	Link 3
Path 2 (Road)	Links 2-5-6	Links 4-5-7
Path 3 (PNR)	Links 2-8-9 OR Links 2-5-9	Links 4-8-10 OR Links 4-5-10

The upper limit of the parking size $\overline{Y}$ is set to be 20 while the lower bound $\underline{Y}$ is set to be 2. The number of population is set to be 6. Common stop criteria of GA is to stop the algorithm either on reaching maximum number iterations or when the best answer does not improve in a certain number of iterations. In this paper, if the value of objective functions does not improve after certain number of generations, the algorithm is terminated. The mutation and crossover rate are set to be 0.25 and 0.45 respectively.

The evaluation procedure includes the traffic assignment problem based on equivalent NCP formulation of the disutility-related multi-class user equilibrium and solves the problem using path-based algorithm adapted from Yin and Ieda's work (Yin and Ieda, 2001). This lower level traffic assignment problem is solved for each newly chromosome.

Figure 2 demonstrates the convergence trend of the GA and provides the minimal value of the objective function for each generation for the special case of β_5 = 1. The optimal P&R scheme and corresponding value of objective function are presented in Table 4. The results account for different consideration in the levels of travel time variations in link 5 (i.e. β_5). As seen, node 6 becomes the optimum choice of parking location while the network link travel times are deterministic. However, as soon as the link 5 travel time becomes stochastic, node 5 is the choice for the P&R facility. Based on parking demand, $\overline{Y}$ = 20 is found to be the optimal parking size in either case.

Figure 2: Convergence trend of GA for β_5 = 1



This study also investigates how the optimal decision of the P&R scheme improves the reliability of the network as a whole. Table 5 presents a comparison between network reliabilities for the worst P&R decision (location and parking size) and the best P&R decision (location and parking size) for different degree of travel time variations perceived on link 5 i.e. for different values of β_5. It should be noted that the values of the network reliability obtained for the worst P&R decision are still higher than the same for cases with no P&R considered (i.e. all auto links). Hence, the network reliability obtained for the worst P&R decision is used as the threshold value of reliability, R_{th} for optimum P&R network design. To cope with the constraint in Equation (4), the reliability of the network must be larger than or equals to this threshold, R_{th}. Values of reliability presented in Table 5 are normalised for the best P&R decision at β_5= 0, i.e. for the deterministic network. As can be seen from the Table, the reliability of the network is greatly improved by optimal decision of P&R facility location and the corresponding capacity. This improvement is evident for different degree of travel time variations in the network as indicated in the Table 5.

Table 4: Optimal P&R decision

Values of β_5	Optimal P&R scheme		
	Parking Location	Parking Size	Travel Disutility (Z)
0	Node 6	20	13033
1	Node 5	20	13633
2	Node 5	20	16409

Table 5: Reliability improvement by upper level network design

Values of β_5	Network reliability for no P&R (all auto links only)	Network reliability for worst P&R decision; also used as R_{th}	Improved network reliability by optimal P&R decision, R
0	0.74	0.78	1.0

1	0.72	0.76	0.96
2	0.64	0.66	0.79

5. Conclusions

This paper has presented a heuristic approach for Park-and-Ride network design in a bi-modal transport network to optimise the reliability of network in terms of the travel disutility within the whole network. A bi-level programming model has been developed to obtain the optimal P&R scheme with minimal expected travel disutility. The upper level model minimises the total travel disutility for the whole network while the lower level is network users' route choice equilibrium problem. The network users, while making decision on the choice of path in the network under non recurrent congestions, were considered to exhibit different risk-taking behaviours depending on the level of variations of their perceived travel time which is referred to as travel disutility. The disutility-related multi-class user equilibrium approach was employed in the lower level problem. An efficient solution algorithm based on the Genetic Algorithm has been proposed to solve the P&R network design problem. The developed model was applied to a small sized example network and it was found that the reliability of the network as a whole is improved with the incorporation of the optimal decision for the P&R scheme in the network. The numerical example presented in this paper is based on only risk-averse commuters. This study is an initiative towards the novel research area of optimising network reliability in P&R network design. A fixed travel demand is considered in this paper. Future work will investigate the impact of possible increasing travel demand for establishing P&R facilities. This study may also be extended for time-dependent or dynamic transport framework. Numerical evaluation will include other commuter behaviours as risk-neutral and risk-seeking.

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