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Genetics of traffic assignment models for strategic transport planning

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Abstract

This paper presents a review and classification of traffic assignment models for strategic transport planning purposes by using concepts analogous to genetics in biology. Traffic assignment models share the same theoretical framework (DNA), but differ in functionality (genes). We argue that all traffic assignment models can be described by two genes. The first gene determines the spatial functionality (unrestricted, capacity restrained, capacity constrained, capacity and storage constrained) described by five spatial interaction assumptions, while the second gene determines the temporal functionality (static, semi-dynamic, dynamic) described by two temporal interaction assumptions. This classification provides a deeper understanding of the often implicit assumptions made in traffic assignment models described in the literature, allows for comparing different models in terms of functionality, and opens the way for developing novel traffic assignment models.

1. Introduction

1.1 Background

Traffic assignment models are used all over the world in strategic (long term) transport planning and project appraisal to forecast future traffic flows and travel times. Road authorities typically apply traditional models on large scale road networks for this purpose. These models were developed in the 1950s and have not changed much since, although solution algorithms have become significantly more efficient. Over the past few decades, there have been several new developments leading to more advanced traffic assignment models that describe flows and travel times more realistically. However, these sophisticated models need significantly more calculation time and memory and are generally more difficult to calibrate.

There exists a wide range of traffic assignment models proposed in the literature, ranging from static to dynamic models and ranging from models that consider free-flow conditions to models that consider congestion with queuing and spillback. These models differ in computational complexity and realism, each making their own simplifying assumptions. However, these assumptions are often implicit. In this paper we aim to disentangle the characteristics of traffic assignment models and explicitly state the assumptions underlying these models. Deeper insights in these assumptions allows a better understanding of the capabilities of each model and the circumstances under which models can be applied.

We narrow the scope of this paper by making the following limiting assumptions: (i) macroscopic description of traffic flow, (ii) within-day equilibrium, (iii) only first order effects are considered, (iv) inelastic travel demand, (v) only a single user class is considered, and (vi) only travel time is considered in route choice. The first three assumptions are made because the focus is on traffic assignment models for strategic transport planning purposes, which in general do not consider mesoscopic or microscopic representations of traffic flows (with possible random components), assume a user-equilibrium solution in order to compare scenarios, and ignore second order effects (such as capacity drop, stop-and-go waves, and hysteresis). The last three assumptions are made to restrict ourselves to the core components of traffic assignment models in which we aim to find a route choice equilibrium with a given travel demand (and do not include departure time choice, mode choice, destination choice, or other travel choices) for a single user class (passenger cars, or passenger car equivalents) considering only travel time (and do not include other generalised cost components). These last three assumptions can be relaxed and are not strictly necessary for our framework, but they allow a more focussed presentation of the concepts in this paper.

1.2 Genetics

In this paper we describe the 'genetics' of traffic assignment models, which allows us to describe and characterise models in a qualitative fashion. Although the various traffic assignment models proposed in the literature may seem very different and sometimes incompatible, they share the same DNA and can be seen as children of the same ancestors having different genes.

In biology, DNA is a blueprint of life that consists of instructions that control the functions of cells. Each species (e.g., humans) shares more or less the same DNA. The building blocks of DNA are called nucleotides, which store genetic information. Genes describe basic functions of living organisms and consist of a specific sequence of nucleotides. The genetic code therefore describes all characteristics of the organism. DNA is inherited from parents through recombination, and evolves through mutation (i.e., genetic variation).

Traffic assignment models can be thought of as being characterised by a genetic code containing model assumptions and genes that describe functionality. Each traffic assignment model for strategic transport planning shares the same theoretical framework (DNA). We identify two different genes, namely one gene that describes spatial interactions, and one gene that describes temporal interactions. These genes are composed of nucleotides that delineate each individual assumption that impacts the functional capability of the model. By combining different temporal and spatial interaction assumptions, different traffic assignment models can be created.

A very capable organism with many positive characteristics is sometimes said to have 'good genes'. Advanced traffic assignment models have 'better' genes than their simpler traditional counterparts. Just like living organisms, traffic assignment models have evolved over time, often by small mutations in one of the underlying assumptions, sometimes by recombination of existing models into a new model. By discovering basic underlying assumptions of each model (genetic code), we can investigate model functionality and limitations, as well as propose improved models. It also allows genetic modifications of existing models to develop novel models.

1.3 Paper outline

In Section 2 we describe the DNA of traffic assignment models, which allows us to classify each traffic assignment model. Sections 3 describes the first gene using five nucleotides that represent the spatial interaction assumptions. Section 4 describe the second gene, consisting of two nucleotides that represent the temporal interaction assumptions. Using these assumptions, Section 5 classifies a selection of traffic assignment models proposed in the literature. Finally, we draw conclusions in Section 6 and state the potential for new model development.

2. DNA of traffic assignment models

In the literature, the main distinction that is often made between models is with respect to temporal interaction assumptions, i.e. whether a model is static or dynamic. Dynamic models are typically seen as superior over static models. However, certain static models are capable or accounting for queues and even spillback while certain dynamic models may not. We therefore need a more elaborate classification of traffic assignment models that describes their characteristics and capabilities.

In this section we propose a unified theoretical framework (DNA) for traffic assignment models.

2.1 Model types as a result of two genes

Traffic assignment models can be classified according to different model types that result from two different genes, namely spatial and temporal interaction assumptions, see Figure 1. Details of these assumptions will be discussed in Sections 3 and 4.

As a result of spatial interaction assumptions (Gene 1), the following model types can be distinguished:

- Unrestricted traffic assignment models;
- Capacity restrained traffic assignment models;
- Capacity constrained traffic assignment models;
- Capacity and storage constrained traffic assignment models.

In unrestricted models, traffic flow is not interrupted and all travellers choose the route with the lowest free-flow travel time. In all other model types, travellers may experience delays due to other vehicles on the road, which affects route choice. These models adopt the notion of some sort of user equilibrium (e.g., Wardrop, 1952) in which individual travellers aim to minimise their (expected) travel time. In capacity restrained models, queues do not exist and traffic flow can exceed road capacity. Capacity constrained models explicitly impose constraints on the traffic flows, resulting in possible queues. However, the number of vehicles in a queue can exceed road storage capacity, such that spillback does not occur. The most capable models are capacity and storage constrained traffic assignment models, which take both physical constraints (capacity and storage) into account. Capabilities of models due to spatial interaction assumptions are further discussed in Section 2.2.

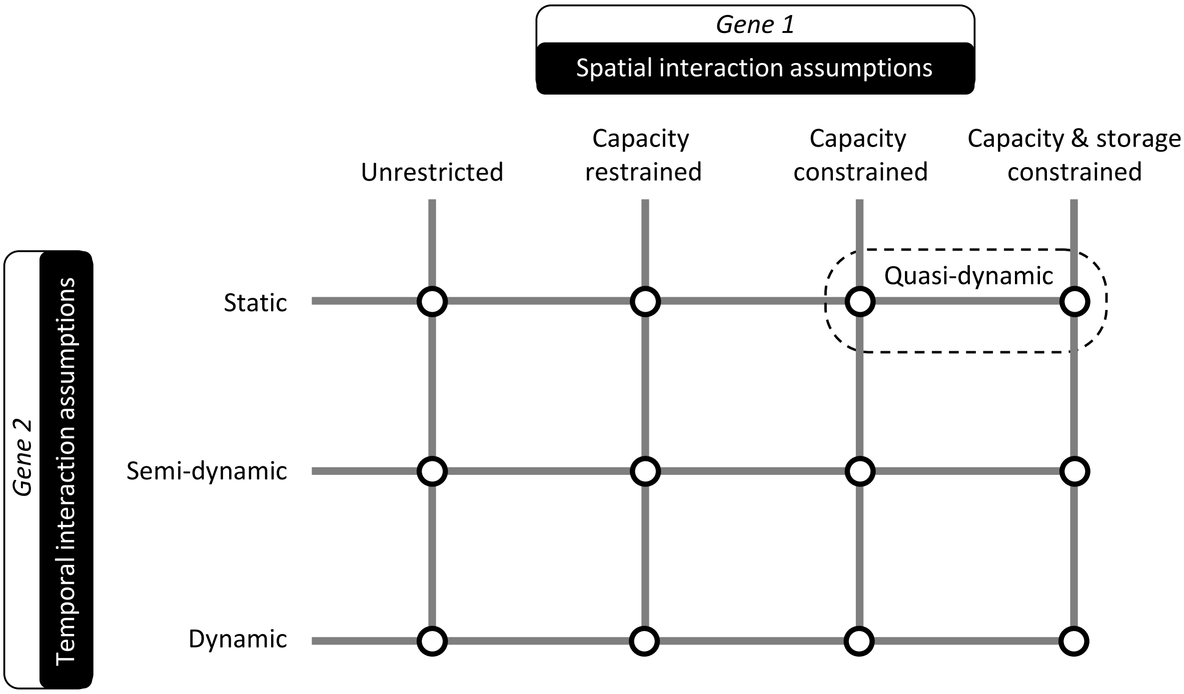
As a result from temporal interaction assumptions (Gene 2), the following model types can be distinguished:

- Static traffic assignment models;
- Semi-dynamic traffic assignment models;
- Dynamic traffic assignment models.

Static models consider a single time period for both route choice and network loading. Traffic flows are considered to be stationary or assumed to represent average traffic flows. Dynamic models consider multiple time periods for route choice and within each of these periods consider multiple (smaller) time periods for network loading. Therefore, dynamic models explicitly account for variations over time in path flows and link flows. Semi-dynamic models consider more than on time period for route choice, but are not completely dynamic. They often only consider a single time period for network loading within each route choice period. Capabilities of models due to temporal interaction assumptions are further discussed in Section 2.3.

Some models are referred to as quasi-dynamic, which can be confusing. Quasi-dynamic models only consider a single time period and do not explicitly model time-varying flows. As such, these models are essentially static. Static models are called ‘quasi-dynamic’ if they impose capacity and/or storage constraints (similar to advanced dynamic models).

Figure 2: DNA of traffic assignment models



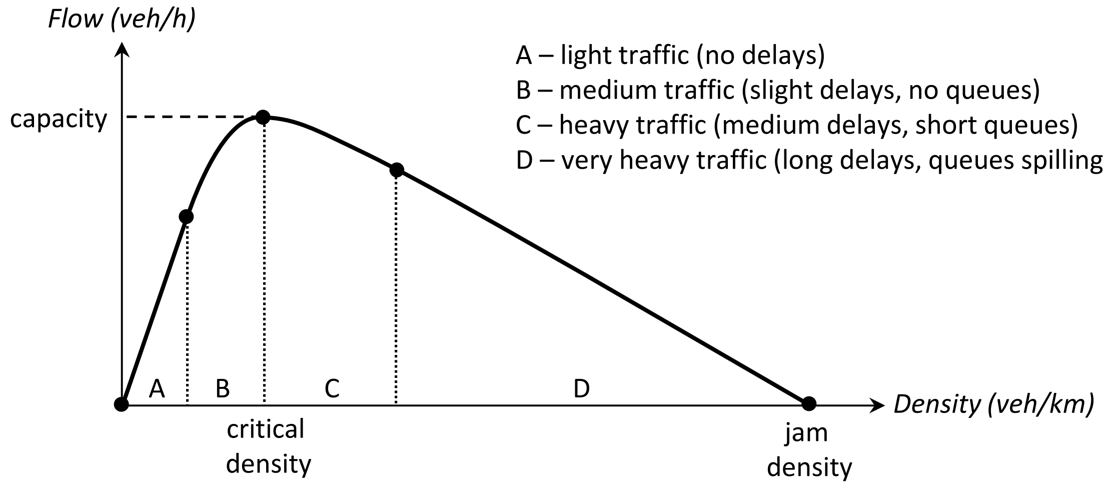
Combinations of spatial and temporal assumptions lead to twelve specific model classes. The simplest model class is a static unrestricted traffic assignment model. This model essentially performs an all-or-nothing assignment. The most elaborate model class is a dynamic capacity and storage constrained (first order) traffic assignment model. Capabilities of the model increases from top left to bottom right in Figure 1.

2.2 Spatial interaction assumptions and model capabilities

The model types associated with different spatial interaction assumptions discussed in Section 2.1 have different capabilities and should ideally only be used in cases where these assumptions are valid. Figure 2 indicates a fundamental diagram describing the relationship between flow and density that can be empirically observed from traffic counts on a road segment. For low densities (indicated by A and B), flow increases with density. These traffic states are called hypocritical states in which no queues appear. High densities (indicated by C and D) are a result of congestion and occur at hypercritical states in which flow decreases with density. Flow is constrained by capacity and density is constrained by the jam density. The maximum number of vehicles that can be stored on a certain road segment can be calculated by multiplying the jam density by the length of the road segment.

Unrestricted models are only suitable for light traffic conditions (A) in which flow increasing linearly with density, indicating that vehicles drive at maximum speed. Capacity restrained models are only suitable for light to medium traffic conditions (A and B) in which the flow does not exceed capacity. These models cannot describe the hypercritical part of the fundamental diagram, such that the flow will continue to increase with increasing density. Capacity constrained models are suitable for light to heavy traffic conditions (A, B, and C) in which short queues can form. These models cannot describe queues longer than the length of the road. Most capable is a capacity and storage constrained model, which can be applied to all traffic conditions (A, B, C, and D). In very heavy traffic, queues can grow longer than the road length and spillback to upstream road segments.

Figure 2: Spatial interaction assumptions and model capabilities



2.3 Temporal interaction assumptions and model capabilities

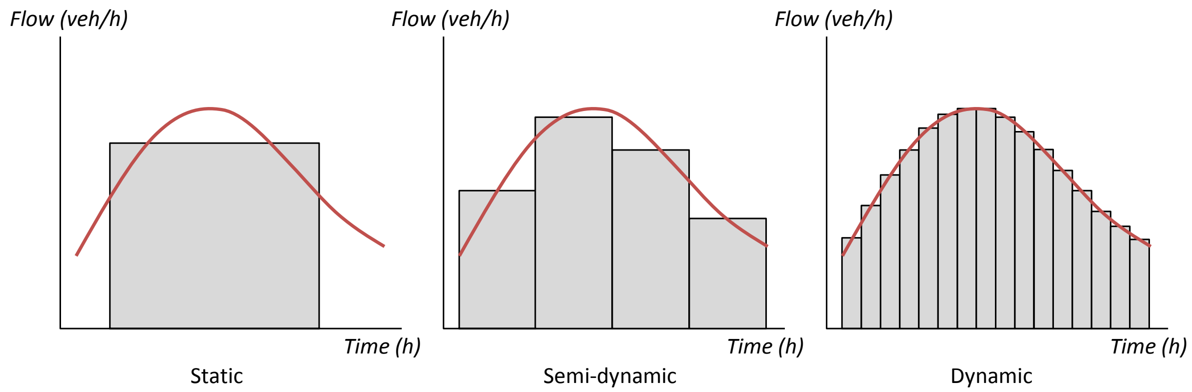
The model types associated with different temporal interaction assumptions discussed in Section 2.1 have different capabilities and should ideally only be used in cases where these assumptions are valid. Figure 3 illustrates how static, semi-dynamic, and dynamic models represent travel demand. The solid red line indicates the actual travel demand for a single origin-destination pair, and the grey bars represent the average demand in the model during each period. The areas of the grey bars (indicating the number of vehicles) are equal to the areas underneath the demand curves.

A static model considers a single time period, typically consisting of an entire peak period (e.g., a three hour period from 6.30am till 9.30am). It assumes that traffic outside this period does not influence flows or travel times in the peak period. Therefore, the time period needs to be sufficiently large such that it contains the entire period during which the demand is above capacity, but also sufficiently small in order to avoid spreading the travel demand too evenly over time (e.g., peak demand may then fall below capacity). Route proportions are assumed stationary during this period. Network loading also considers a single time period in which all traffic reaches the destination and all link flows should be interpreted as average flows during this period.

In a semi-dynamic model multiple time periods are considered (e.g., one hour time slices, such as periods 6-7am, 7-8am, 8-9am, and 9-10am). It can be seen as a sequence of static models, however it takes residual traffic from a previous period (such as vehicles in a queue) into account. As such, semi-dynamic models are better capable of describing variations in travel demand and the interactions of vehicles across time periods. Route proportions are assumed stationary during each time period. Network loading within each time period is usually done in a simple fashion similar to a static model with the limitation that vehicles cannot be propagated more than the duration of each period. In other words, vehicles may not reach their destination within a single time period and may be transferred to the next time period.

Dynamic models are the most capable models and usually consider many smaller time periods (e.g., time slices of 15 minutes). This allows dynamic models to more accurately represent time-varying travel demand. Route proportions are again assumed stationary during each time period. Network loading is much more sophisticated and similar to simulation models consider small time steps (e.g., 1 second) in which vehicles are propagated on the network. Compared to static and semi-dynamic models, dynamic models are able to describe more detailed interactions between vehicles within each time period.

Figure 3: Temporal interaction assumptions and model capabilities



3. Gene 1: Spatial interaction assumptions

The first gene represents the spatial interaction assumptions, which describe how traffic flows spatially interact and directly impact the realism of the model (see also Figure 1). These spatial interactions are a combination of assumptions on the path level (route choice behaviour), the link level (shape of the fundamental diagram, capacity and storage constraints), and the node level (turn flow restrictions yielding turn reduction factors). Each of these spatial interactions have been analysed mostly separately in the literature and can be calibrated empirically.

The five specific assumptions (nucleotides) within this gene are summarised in Table 1 and are discussed in more detail in the following subsections, where level refers to the spatial level at which interactions are described. The spatial interaction assumptions of a traffic assignment model can be indicated using a sequence of letters representing the genetic code. For example, the traditional static traffic assignment model proposed in Beckmann et al. (1956) can be described by the following code for Gene 1: (F,P)(C,N)(U,U)(U)(N). The most sophisticated and capable model is defined by genetic code (B,I)(C,C),(C,C),(C)(F).

Table 1: Genetic code for Gene 1 (spatial interaction assumptions)

Nucleotide	Level	Code	Explanation
Route choice behaviour	Path	F, B	Fully / Boundedly rational behaviour
		P, I	Perfect / Imperfect information
Shape fundamental diagram	Link	L, P, Q, C	Linear / Piecewise linear / Quadratic / Concave hypocritical branch
		L, P, Q, C, H, V, N	Linear / Piecewise linear / Quadratic / Concave / Horizontal / Vertical / Not available hypercritical branch
Capacity constraints	Link	U, C	Unconstrained / Constrained inflow capacity
		U, C	Unconstrained / Constrained outflow capacity
Storage constraints	Link	U, C	Unconstrained / Constrained storage
Turn flow restrictions	Node	F, O, N	First order / Other / No restrictions

3.1 Nucleotide 1 – Route choice behaviour

Each traffic assignment model makes assumptions regarding route choice. Often it is assumed that travellers have perfect knowledge regarding travel times and are fully rational, i.e. (F,P) route choice, leading to an assignment model that aims to find a deterministic user equilibrium. If travellers are assumed to have perception errors when making route choice decisions (i.e., imperfect knowledge), a stochastic user equilibrium can be adopted. For example a logit model (Chen, 1999), a C-logit model (Zhou et al., 2012), or a weibit model (Castillo et al., 2008; Kitthamkesorn and Chen, 2013) can be used in order to describe (P,I) route choice. Others assume that travellers are boundedly rational, in which drivers are assumed not to change route until the cost savings exceed a certain threshold (Han et al., 2014), resulting in a (B,F) route choice model.

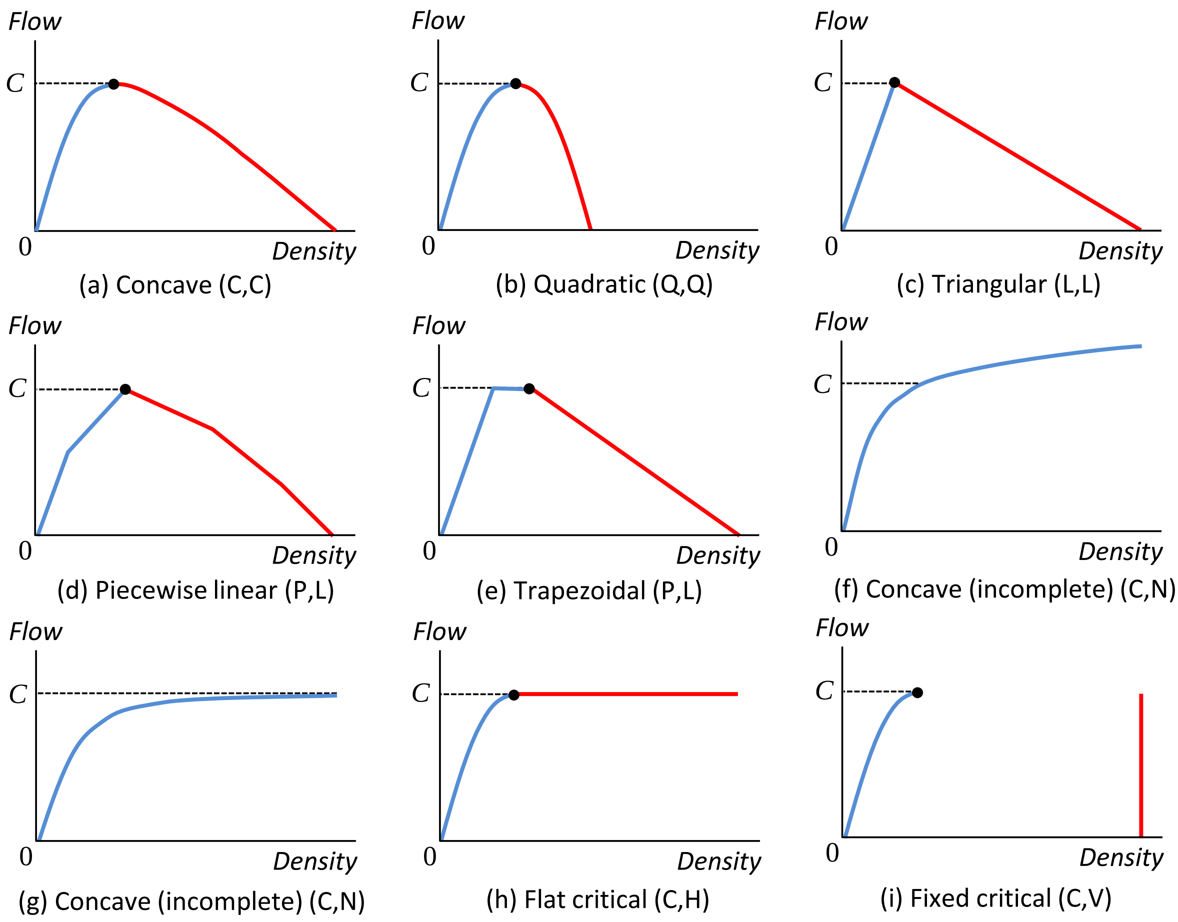
3.2 Nucleotide 2 – Shape of the fundamental diagram

The shape of the fundamental diagram plays an important role in traffic flow theory and different shapes lead to different traffic patterns on a link. A fundamental diagram is often directly estimated from cross-sectional traffic counts and space-mean speeds. It is generally defined by an increasing concave hypocritical branch (for densities lower than the critical density, indicated in blue in Figure 4(a), consistent with traffic conditions A and B in Figure 2) and a decreasing concave hypercritical branch (for densities higher than the critical density, indicated in red in Figure 4(a), consistent with traffic conditions C and D in Figure 2). Such a general function can be indicated by (C,C) using the coding from Table 1.

The first fundamental diagram is described in Greenshields (1935). He proposed a linear relationship between speed and density, which results in a quadratic fundamental diagram (Q,Q), see Figure 4(b). Such a symmetric fundamental diagram may describe hypocritical traffic conditions quite accurately, but performs poorly for hypercritical states. A popular choice in traffic flow theory due to computational advantages has been an asymmetric triangular fundamental diagram (L,L) (Newell, 1993) as shown in Figure 4(c). While a linear relationship in the hypercritical branch is often considered sufficiently realistic, a linear relationship in the hypocritical branch is less realistic (since it assumes that the speed at capacity is equal to the maximum speed). Therefore, piecewise linear fundamental diagrams (P,P) as shown in Figure 4(d) have been proposed (e.g., Yperman, 2007), which maintain many computational benefits. A special case of such a piecewise linear fundamental diagram is the trapezoidal fundamental diagram (Daganzo, 1994) shown in Figure 4(e).

Although fundamental diagrams have been used extensively in several dynamic traffic assignment models, static models have mainly relied on link performance functions (also called volume-delay functions), which describe the relationship between link travel time and link flow (volume) or between the speed and flow. The most well-known link performance function is the BPR link performance function (Bureau of Public Road, 1964). This function is plotted in Figure 4(f). Two things can be observed from this (C,N) shape. First, the BPR function only contains the hypocritical branch of the fundamental diagram and ignores the hypercritical branch. Secondly, the hypocritical branch increases beyond the capacity C. Davidson (1966) proposed an alternative to the BPR function in which the travel times goes to infinity as the flow approaches capacity, such that the hypocritical branch of the fundamental diagram has a horizontal asymptote at capacity as shown in Figure 4(g). However, this provides computational problems when flow reaches capacity. While others have discussed modifications to eliminate these problems (e.g., Daganzo, 1977; Taylor, 1984), it is clear that link performance functions do not realistically describe the relationship between traffic flow and density at higher densities.

Figure 4: Shapes of the fundamental diagram



Diagrams shown in Figure 4(a)-(e) result in models with physical queues since they have a downward sloping hypercritical branch, while the diagrams in Figure 4(f)-(g) do not result in any queues since the hypercritical branch is absent. Other shapes of the hypercritical branch of the fundamental diagram have been proposed that result in specific types of queues. A fundamental diagram with a horizontal hypercritical branch as shown in Figure 4(h) is consistent with a model with vertical (non-physical) queues, while a vertical hypercritical branch as shown in Figure 4(i) yields a model with horizontal (physical) queues in which all queues are assumed to have a fixed queuing density, either equal to the jam density (leading to very compact queues) or some other fixed queuing density (Bliemer, 2007).

3.3 Nucleotide 3 – Capacity constraints

Some models consider capacity constraints, while others assume no upper bounds on traffic flows. In case no constraints on the link entrance and exit flows are assumed, i.e., (U,U), no queues build up. This is consistent with fundamental diagrams in Figure 4(f). When considering both link entrance and exit capacity constraints in the (C,C) case, they are typically set to the physical link capacity C. In this case, residual queues will form upstream the bottleneck link. Some models consider (U,C), in which only link exit capacities are considered. In other words, flow is not restricted to flow in, but is restricted when flowing out. Such an assumption leads in some situations to queues inside the bottleneck. Finally, models can also only consider (C,U), which consider link entrance capacities and no explicit outflow constraints.

3.4 Nucleotide 4 – Storage constraints

When the number of vehicles in a queue exceeds the available link storage, the queue will spill back to upstream links. The maximum number of vehicles that can be stored on a link is equal to the jam density times the link length. Some models do not consider spillback, thereby implicitly assuming no storage constraints (U). This essentially means an infinite jam density, which is consistent with the fundamental diagrams presented in Figure 4(h). Models that take storage constraints into account (C) have a finite jam density, consistent with the fundamental diagrams in Figures 4(a)-(e) and 4(i).

3.5 Nucleotide 5 – Turn flow restrictions

Given that queues and travel time delays are mainly determined by node models, it is perhaps surprising to see that almost all static traffic assignment models and some dynamic models completely lack a node model description. In case there are no capacity constraints on the link entrance or exit flows, queues will never occur and hence a node model can often be omitted (N).

In the presence of capacity constraints, node models determine the turn flows at intersections, merges, and diverges. Tampère et al. (2011) describes requirements for a first order node model for a node with any number of incoming and outgoing links. These requirements include flow maximisation, non-negativity, satisfying demand and supply constraints, satisfying the conservation of turn fractions (CTF) and the invariance principle (see Lebacque and Khoshyaran, 2005). Merge constraints that follow the capacity based weighted queuing rule (Ni and Leonard II, 2005) satisfy the invariance principle, in which the outflow rates are capacity proportional in case both in-links are congested. On the other hand, merge constraints that satisfy the fair merging rule (Jin and Zhang, 2003) in which inflow rates are demand proportional, do not satisfy the invariance principle.

Bliemer (2007) combines a first-in-first-out diverging rule and the fair merging rule into a closed form demand proportional model for general cross nodes. Several node models for general nodes have been proposed in the last decade (e.g., Jin and Zhang, 2004; Jin, 2012a; Jin, 2012b), none of them satisfy both CTF and the invariance principle and are therefore classified under other turn flow restrictions (O).

Recently, models have been proposed that satisfy all requirements for first order node models, including CTF and the invariance principle, see e.g. Tampère et al. (2011), Flötteröd and Rohde (2011) and Gibb (2011). These models mainly differ in the merge constraints. Recently, Smits et al. (2015) described a family of macroscopic first order node models (F), which includes the three node models mentioned above.

4. Gene 2: Temporal interaction assumptions

In this section we consider temporal interaction assumptions identified in the second gene that determines whether a model is static, semi-dynamic, or dynamic. These assumptions consider interactions within time periods (forward and backward wave speeds) as well as across time periods (residual traffic transfer). Each of these assumptions aims to remove or simplify time dynamics within the model, in particular in the network loading model.

The two specific assumptions (nucleotides) within this gene are summarised in Table 2 and are discussed in more detail in the following subsections, where level refers to the temporal level (within-period or across periods) at which the interactions are described. The temporal interaction assumptions for the traditional static traffic assignment model proposed in Beckmann et al. (1956) can be described by the following code for Gene 2: (I,N)(N). The most sophisticated and capable model is defined by genetic code (K,K)(T).

Table 2: Genetic code for Gene 2 (temporal interaction assumptions)

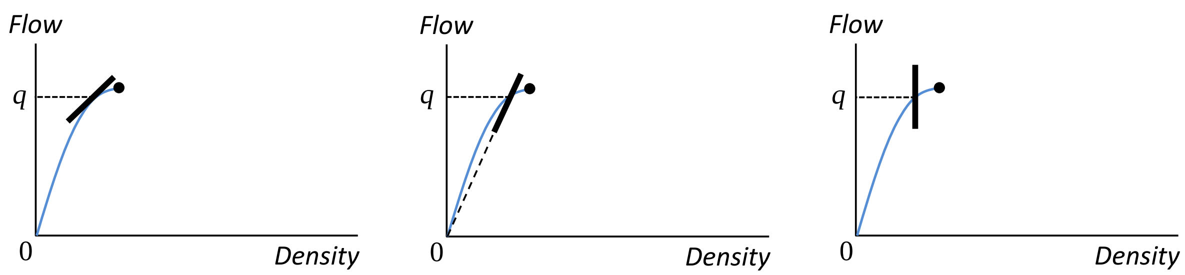
Nucleotide	Level	Code	Explanation
Wave speeds	Within	K, V, I	Kinematic / Vehicular / Infinite forward speeds
		K, I, Z, N	Kinematic / Infinite / Zero / Not available backward speeds
Residual traffic transfer	Across	T, N	Transfer / No transfer of vehicles

4.1 Nucleotide 6 – Wave speeds

Temporal interactions on a network are described by forward and backward wave speeds. In order to determine the vehicles flows in the network, we need to know the speed at which traffic states propagate through the network, which we refer to as forward wave speeds. In the first order LWR model (Lighthill and Whitham, 1955; Richards, 1956), traffic conditions travel at the kinematic wave speed (K) equal to the slope of the hypocritical branch of the fundamental diagram as shown in Figure 5(a) for traffic flow rate q. It is important to realise that the speeds at which traffic states propagate and the speeds at which vehicles are propagated through the network are in general not the same. On case of a concave hypocritical branch, the vehicular speed (V) –which is equal to the flow divided by the density and hence equal to the slope of the line connecting the origin to the traffic state as shown in Figure 5(b) – is always larger than (or equal to) the kinematic wave speed. Only if the hypocritical branch is linear, these speeds are equal. More recent dynamic models consider kinematic wave speeds, but especially earlier dynamic models and semi-dynamic models consider vehicular speeds.

All static models simplify the within-period interactions by use infinite forward wave speeds (I) in which traffic flows instantaneously propagate through the network and reach their destination within the single period. This situation is illustrated in Figure 5(c). This assumption effectively removes the necessity to track traffic states or vehicles over time.

Figure 5: Forward wave speeds

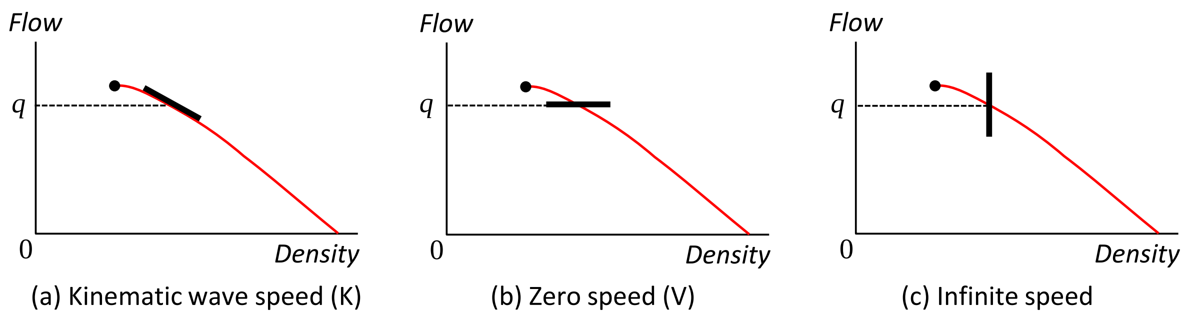


(a) Kinematic wave speed (K)   (b) Vehicular speed (V)   (c) Infinite speed

Backward waves track how traffic states propagate backwards on a road segment, and are responsible for queue build up and possible spillback to upstream links. In the LWR model traffic conditions travel at the (negative) kinematic wave speed (K) equal to the slope of the hypercritical branch of the fundamental diagram as shown in Figure 6(a) for traffic state q. Similar to forward waves, it requires a dynamic model to explicitly deal with the effects of such backward kinematic waves over time.

Most static models do not consider a hypercritical branch in the fundamental diagram such that backward wave speeds are not available (N). Only quasi-dynamic models with capacity constraints can describe queues in static models by implicitly assuming the presence of a hypercritical branch. Two different temporal assumptions regarding backward waves can be made in order to remove the time dimension in a static model. The most widely adopted implicit assumption in quasi-dynamic models is setting backward wave speeds equal to zero (Z) as shown in Figure 6(b). In this case, traffic conditions never move backwards, which means vertical non-physical queues and no spillback. The zero speed assumption is consistent with fundamental diagrams of the shape shown in Figure 4(h). The other assumption that removes time from the model is considering a (negative) infinite speed depicted in Figure 6(c), which allows the model to describe spillback when the number of vehicles in the queue exceeds the storage capacity. Note that an infinite speed does not mean that queues build up indefinitely, since the length of the queue is constrained by the number of vehicles in the queue. The fundamental diagram in Figure 4(i) is consistent with the infinite speed assumption.

Figure 6: Backward wave speeds



4.3 Nucleotide 7 – Residual traffic transfer

Residual traffic at the end of a time period results when vehicles are not able to reach their final destination within the considered time period (or the smaller network loading time step). These residual vehicles are either (i) in a residual queue due to a bottleneck downstream, or (ii) simply are not able to reach their final destination because the travel time to reach the destination is longer than the considered time period. This residual traffic influences traffic flows and travel times in the next time period. This dependency of traffic across time periods can be eliminated by assuming that any residual traffic has no impact on the next time period, in other words, assuming that the network is empty at the beginning of each time period.

Dynamic models transfer all traffic from period to period (T), thereby describing the full temporal interactions across time periods. Static models consider no residual traffic transfer (N), such that each time period is modelled independently. Especially when modelling short time periods in a congested network, this independence assumption will not be valid. The main difference between static and semi-dynamic models is that the latter does assume residual traffic transfer between time periods as discussed in Section 2.3.

5. Classification of existing traffic assignment models

Many static and dynamic models have been proposed in the literature that we can classify using the seven nucleotides in the two genes. First we will discuss several static models, and then some dynamic models as listed in Table 3. It should be stressed that this list is by no means intended to be complete, but rather to provide typical examples of certain types of traffic assignment models. We omit semi-dynamic models because they are scarce and typically formulated as procedures and algorithms rather than mathematical problems, e.g., Van Vliet (1982) and Davidson et al. (2011), which makes them difficult to accurately classify.

5.1 Static models

The traffic assignment model with deterministic route choice proposed by Beckmann et al. (1956) is the most widely adopted static model formulation, however the assumed spatial interactions are not very realistic. It implicitly assumes a fundamental diagram as in Figure 4(f) in which the hypercritical branch is missing. Further, no capacity constraints or storage constraints nor turn flow restrictions are considered, resulting in link traffic flows exceeding physical link capacities. In order words, this model is only appropriate for uncongested conditions without queues building up.

Fisk (1980) extended the unconstrained static traffic assignment model to include stochastic route choice based on the multinomial logit model. Zhou et al. (2012) further extended this model to taken path overlap into account consistent with the C-logit model. Building on Smith (1987), Bell (1995) formulated a model with logit based route choice and capacity constraints.

Bifulco and Crisalli (1998) were among the first to propose a model that explicitly describes residual queues. Stochastic logit based route choice is considered. They implicitly assume a fundamental diagram as in Figure 4(h) in which fixed link exit capacity constraints are taken into account. Moreover, neither link storage capacity constraints nor turn flow restrictions are considered. Backward kinematic wave speeds are implicitly assumed to be zero, such that the resulting model describes vertical point queues without spillback. Implicitly taking the CTF node constraints into account, they propose turn reduction factors equal to capacity divided by sending flow. Similar models that explicitly describe residual queues have been proposed by Lam and Zhang (2000) and Smith (2013) in which deterministic route choice is assumed.

The first static traffic assignment model to consider a finite link storage capacity constraints is proposed by Smith et al. (2013). Similar to previous models with residual queues, only link exit capacity constraints are considered, while link entrance constraints and other turn restrictions are mostly ignored (although merges and diverges are briefly discussed). They implicitly assume a fundamental diagram similar to Figure 4(i), which leads to compact horizontal queues (with densities equal to the jam density) and spillback when the number of vehicles in the queue exceeds the link storage capacity. Their model is restricted to a specific hypothetical network and not generally applicable to transport networks that include more complex merges and diverges.

Bliemer et al. (2014b) are the first to include a first order node model that explicitly describes turn flow restrictions into a static traffic assignment model with residual queues for general transport networks. Their model can be seen as an extension of the model of Bifulco and Crisalli (1998). Bliemer et al. (2014b) further derive for the first time a path travel time function consistent with travel time calculations in dynamic models.

Bliemer et al. (2014a) have derived static traffic assignment models that further generalise the model in Bliemer et al. (2014b) taking path overlap into account and considering a general concave form of the uncongested branch of the fundamental diagram instead of a linear functional form. They further derive a new model with physical horizontal queues and spillback consistent with a general concave fundamental diagram. These novel models represent static versions of dynamic traffic assignment models that maintain all spatial interactions.

5.2 Dynamic models

One of the earlier dynamic traffic assignment models is described in Janson (1991), which can be seen as a direct extension of the traditional static traffic assignment model. The underlying SIAs are a fundamental diagram as in Figure 4(f), without considering a hypercritical branch, no capacity constraints, no storage constraints, and no turn flow restrictions. Link travel times are a function of only the link flows in the current time period. Hence, travel times are separable and the problem can be formulated as an optimisation problem. Chen and Hsueh (1998) consider a similar model, but the link travel times are considered to be a function of all previous link inflows, leading to a non-separable link travel time function with asymmetric Jacobians, and resulting in a variational inequality problem formulation. Both models assume that forward wave speeds are equal to the vehicular speeds. As Ran and Boyce (1996) point out, this assumes stationary travel times and ignores platoon dispersion or concentration. Astarita (1996) derives a more general relationship that is consistent with kinematic wave theory and has also been used the (multiclass) dynamic models in Bliemer and Bovy (2003) and generalised in Chabini (2001).

The model proposed in Bliemer (2007) was one of the first analytical dynamic traffic assignment models to include capacity constraints, link storage constraints, and turn flow restrictions. The implicitly assumed fundamental diagram is consistent with Figure 4(i) with infinite backward wave speeds at a queuing density lower than the jam density. Queues spill back to upstream links if a fixed and finite link storage capacity is exceeded. CTF constraints and the fair merging rule was used, resulting in demand proportional turn flows not consistent with a first order node model (since it violates the invariance principle). Yperman (2007) proposed a first order link model with physical queues, under the strict assumption of a triangular fundamental diagram (see Figure 4(c)). The node model describing turn flow restrictions is similar as in Bliemer (2007). Gentile (2010) relaxed the assumption of a triangular fundamental diagram to allow for any concave fundamental diagram (see Figure 4(a)). The models of Yperman (2007) and Gentile (2010) allow proper forward and backward kinematic wave speeds consistent with traffic flow theory. Yperman (2007) and Gentile (2010) focus on the dynamic network loading model, while mostly ignoring the route choice aspect.

More recently, Friesz et al. (2013) derives a similar model to Gentile (2010), but implicitly assumes a horizontal hypercritical branch in the fundamental diagram (see Figure 4(h)) and assumes that storage capacities are infinite such that spillback does not occur. Further, CTF constraints are considered.

Bliemer et al. (2014a) describes a fairly general first order dynamic traffic assignment model that incorporates general route choice behavioural rules, a general concave fundamental diagram, link entrance and exit capacities, finite link storage capacities, and turn flow restrictions following a first order node model. This results in a state-of-the-art dynamic traffic assignment model that describes physical horizontal queues and spillback consistent with first order traffic flow theory.

Table 3: Overview of assumptions made in different traffic assignment models proposed in the literature

	Gene 1: Spatial interaction assumptions					Gene 2: Temporal interaction assumptions	
	route choice behaviour	fundamental diagram	capacity constraints	storage constraints	turn flow restrictions	wave speeds	residual traffic transfer
Static models							
Beckmann et al. (1956)	R,P	C,N	U,U	U	N	I,N	N
Fisk (1980)	R,I	C,N	U,U	U	N	I,N	N
Bell (1995)	R,I	L,H	U,C	U	N	I,Z	N
Bifulco and Crisalli (1998)	R,I	C,H	U,C	U	O	I,Z	N
Lam and Zhang (2000)	R,P	C.H	U,C	U	O	I,Z	N
Zhou et al. (2012)	R,I	C,N	U,U	U	N	I,N	N
Smith (2013)	R,P	L,H	U,C	U	O	I,Z	N
Smith et al. (2013)	R,P	C,V	U,C	C	O	I,I	N
Bliemer et al. (2014b)	R,I	L,C	C,C	U	F	I,Z	N
Bliemer et al. (2014a)	R,I	C,C	C,C	C	F	I,I	N
Dynamic models							
Janson (1991)	R,P	C,N	U,U	U	N	V,N	T
Chen and Hsueh (1998)	R,P	C,N	U,U	U	N	V,N	T
Chabini (2001)	R,P	C,N	U,U	U	N	K,N	T
Bliemer and Bovy (2003)	R,P	C,N	U,U	U	N	K,N	T
Bliemer (2007)	--	C,V	C,C	C	O	K,I	T
Yperman (2007)	--	L,L	C,C	C	O	K,K	T
Gentile (2010)	R,P	C,C	C,C	C	O	K,K	T
Friesz et al. (2013)	R,P	C,?	U,C	U	O	K,Z	T
Bliemer et al. (2014a)	R,I	C,C	C,C	C	F	K,K	T

6. Discussion and conclusions

In this paper we have presented a theoretical framework with which we can classify traffic assignment models for strategic transport planning. This framework is described in terms of DNA with two genes consisting of five spatial interaction assumptions and two spatial interaction assumptions.

As a special case, the traditional static traffic assignment model proposed by Beckmann et al. (1956) can be derived by assuming (i) rational travellers with perfect knowledge, (ii) a concave hypocritical part and no hypercritical part of the fundamental diagram, (iii) no flow capacity constraints, (iv) no storage capacity constraints, (v) no turn flow restrictions, (vi) infinite forward wave speeds and no backward waves, and (vii) no residual traffic transfer. These very strict assumptions limit the applicability of the model. Although this model is often applied to heavily congested networks, this model is actually only appropriate for uncongested traffic conditions (A and B as indicated in Figure 2).

The quasi-dynamic models that are developed in order to extend the capabilities and realism of static models in congested situations increasingly share commonalities with the spatial interaction assumptions made in advanced dynamic models. As shown in Bliemer et al. (2014a), novel quasi-dynamic models can be derived from advanced dynamic models by simply using static temporal interactions assumptions. Therefore, the framework described in this paper is not only useful for classifying models, but also for developing new models.

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References

Astarita, V. (1996). A continuous time link model for dynamic network loading based on travel time function. Transportation and Traffic Theory. Proceedings of the 13th International Symposium on Transportation and Traffic Theory, pp. 79–101.

Beckmann, M.J., C.B. McGuire, and C.B. Winsten (1956) Studies in economics of transportation, Yale University Press, New Haven CT, USA.

Bell, M.G.H. (1995) Stochastic user equilibrium in networks with queues. Transportation Research Part B, Vol. 29, No. 2, pp. 115–112.

Bifulco, G., and Crisalli, U. (1998). Stochastic user equilibrium and link capacity constraints: formulation and theoretical evidencies. In Proceedings of the European Transport Conference. pp. 85–96.

Bliemer, M.C.J. and P.H.L. Bovy (2003) Quasi-variational inequality formulation of the multiclassdynamic traffic assignment problem. Transportation Research Part B, Vol. 37, pp. 501-519.

Bliemer, M.C.J. (2007) Dynamic queuing and spillback in an analytical multiclass dynamic network loading model. Transportation Research Record: Journal of the Transportation Research Board, No. 2029, pp. 14-21.

Bliemer, M.C.J., Raadsen, M.P.H., Brederode, L.J.N., Bell, M.G.H., Wismans, LJ.J. (2014a) Unified framework for traffic assignment: deriving static and quasi-dynamic models consistent with general first order dynamic traffic assignment models. Proceedings of the 5th International Symposium on Dynamic Traffic Assignment, Salerno, Italy.

Bliemer, M.C.J., Raadsen M.P.H., Smits E.S., Zhou B., Bell M.G.H. (2014b) Quasi-dynamic static traffic assignment with residual point queues incorporating a first order node model. Transportation Research Part B, Vol. 68, pp. 363-384.

Bureau of Public Roads (1964) Traffic Assignment Manual. U.S. Dept. of Commerce, Urban Planning Division, Washington D.C., USA.

Chabini, I. (2001). Analytical dynamic network loading problem formulation, solution algorithms, and computer implementations. Transportation Research Record Vol. 1771, Transportation Research Board, Washington, D.C., pp. 191–200.

Chen, H.K., Hsueh, C.F. (1998). A model and an algorithm for the dynamic user optimal route choice problem. Transportation Research Part B. Methodological. Vol. 32, No. 3, pp 219-234.

Daganzo, C.F. (1977) On the traffic assignment problem with flow dependent costs—II. Transportation Research, Vol. 11, pp. 439–441.

Daganzo, C. (1994) The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory, Transportation Research Part B, Vol. 28, No. 4, pp. 269–87.

Davidson, K.B. (1966) A flow travel-time relationship for use in transport planning. Proceedings of the 3rd Australian Road Research Board Conference, Vol. 8, No. 1, pp. 32-35.

Davidson, P., Thomas, A., & Teye-Ali, C. (2011). Clocktime assignment: a new mesoscopic junction delay highway assignment approach to continuously assign traffic over the whole day. In European Transport Conference Proceedings.

Fisk, C. (1980). Some developments in equilibrium traffic assignment. Transportation Research Part B: Methodological, Vol. 14 No. 3, pp. 243–255.

Flötteröd, G., and J. Rohde (2011) Operational macroscopic modeling of complex urban road intersections. Transportation Research Part B, Vol. 45, No. 6, pp. 903-922.

Friesz, T.L., K. Han, P.A. Neto, A. Meimand, and T. Yao (2013) Dynamic user equilibrium based on a hydrodynamic model. Transportation Research Part B, Vol. 47, pp. 102-126.

Gentile G. (2010) The General Link Transmission Model for Dynamic Network Loading and a comparison with the DUE algorithm, in L.G.H. Immers, C.M.J. Tampère, and F. Viti (eds) New developments in transport planning: advances in Dynamic Traffic Assignment. Transport Economics, Management and Policy Series, Edward Elgar Publishing, MA, USA.

Gibb, J. (2011) Model of traffic flow capacity constraint through nodes for dynamic network loading with queue spillback. Transportation Research Record: Journal of the Transportation Research Board, No. 2263, pp. 113-122.

Greenshields, B.D. (1935) A study of traffic capacity. Highway Research Board Proceedings, Vol. 14, pp. 448–477.

Han, K., W.Y. Szeto, and T.L. Friesz (2014) Formulation, existence, and computation of simultaneous route-and-departure choice bounded rationality dynamic user equilibrium with fixed or endogenous user tolerance. Working paper.

Janson, B.N. (1991) Dynamic traffic assignment for urban road networks. Transportation Research Part B, Vol. 25, No. 2/3, pp. 143-161.

Jin, W.-L. (2012a) The traffic statics problem in a road network. Transportation Research Part B, Vol. 46, pp. 1360-1373.

Jin, W.-L. (2012b) A kinematic wave theory of multi-commodity network traffic flow. Transportation Research Part B, Vol. 46, pp. 1000-1022.

Jin, W.-L., and H.M. Zhang (2003) On the distribution schemes for determining flows through a merge. Transportation Research Part B, Vol. 37, No. 6 , pp. 521–540.

Jin, W.L. and H.M. Zhang (2004) Multicommodity kinematic wave simulation model for network traffic flow. Transportation Research Record: Journal of the Transportation Research Board, No. 1883, pp. 59-67.

Lam, W., and Zhang, Y. (2000). Capacity-constrained traffic assignment in networks with residual queues. Journal of Transportation Engineering, (April), 121–128.

Larsson, T., and M. Patriksson (1994) A class of gap functions for variational inequalities. Mathematical Programming, Vol. 64, pp. 53-79.

Lebacque, J.P., and M.M. Khoshyaran (2005) First-order macroscopic traffic flow models: Intersection modeling, network modeling. Proceedings of the 16th International Symposium on Transportation and Traffic Theory (ISTTT), pp. 365-386.

Lighthill, M. J., and Whitham, G. B. (1955). On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, Vol. 229, pp. 317–345.

Newell, G.F. (1993) A simplified theory of kinematic waves in highway traffic, Part I: General theory, Part II: Queuing at freeway bottlenecks, Part III: Multi-destination flows, Transportation Research Part B, Vol. 27, pp. 281-313.

Ni, D., and J.D. Leonard II (2005) A simplified kinematic wave model at a merge bottleneck. Applied Mathematical Modelling, Vol. 29, pp. 1054-1072.

Ran, B., and Boyce, D. (1996) Modeling dynamic transportation networks: an intelligent transportation system oriented approach. Springer-Verlag, Heidelberg.

Richards, P.I. (1956). Shock Waves on the Highway. Operations Research, Vol. 4,pp. 42–51.

Smith, M. J. (1987). Traffic control and traffic assignment in a signal-controlled network with queueing. In N. H. Gartner & Wilson M. (Eds.), Proceedings of the Tenth International Symposium on Transportation and Traffic Theory (pp. 61–68). Elsevier.

Smith, M. J. (2013). A link-based elastic demand equilibrium model with capacity constraints and queueing delays. Transportation Research Part C: Emerging Technologies, 29, 131–147.

Smith, M. J., Huang, W., and Viti, F. (2013) Equilibrium in capacitated network models with queueing delays, queue-storage, blocking back and control. Procedia – Social and Behavioral Siences, Vol. 80, pp. 860-879.

Smits, E.S., Bliemer, M.C.J., Pel, A.J., Van Arem, B. (2015) A family of macroscopic node models. Transportation Research Part B, Vol. 74, pp. 20-39.

Tampère, C.M.J. , Corthout, R., D. Cattrysse and L.H. Immers (2011) A generic class of first order node models for dynamic macroscopic simulation of traffic flows, Transportation Research Part B, Vol. 45, pp. 289-309.

Taylor, M.A.P. (1984) A note on using Davidson’s function in equilibrium assignment. Transportation Research Part B, Vol. 18, No. 3, pp. 181-199.

Van Vliet, D. (1982). SATURN: a modern assignment model. Traffic Engineering and Control, 23, 575–581.

Wardrop, J.G. (1952) Some theoretical aspects of road traffic research. Proceedings of the Institute of Civil Engineers, Part II, pp. 325-378.

Yperman, I. (2007) The Link Transmission Model for Dynamic Network Loading. PhD Thesis, Katholieke Universiteit Leuven, Belgium.

Zhou, Z., A. Chen, and S. Bekhor (2012) C-Logit Stochastic User Equilibrium Model: Formulations and Solution Algorithm. Transportmetrica, Vol. 8, No. 1, pp. 17–41.