Capacity and Delay Analysis of Urban Links with Mixed Autonomous and Human-Driven Vehicles

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Abstract. *This paper investigates the traffic flow characteristics of mixed stream of autonomous and human-driven vehicles. The proposed model aims at understanding the fundamental properties of mixed flow of human-driven (N) and autonomous vehicles (AV) such as headway, capacity, and delay at signalized intersections. This is challenging because of intrinsic differences between longitudinal driving characteristics of these two types of vehicles and the convoluted dynamics of car following situation within various combinations of AV and human-driven vehicles. The expected headway of the mixed flow is determined based on the penetration rate of AV and the headways between two successive AV-AV, AV-N, N-AV, and N-N. Furthermore, the upper and lower bounds of mixed flow headway are presented. The theoretical headway is validated by microsimulation data. The estimated headways are then incorporated to derive the delay of a mixed flow at a signalized 2-lane link. Four combinations of (i) mixed lanes, (ii) dedicated lanes for AV and human-driven vehicles, (iii) one mixed lane and one AV dedicated lane, and (iv) one mixed lane and one human-driven vehicle dedicated lane are considered. The results demonstrate the performance of the four lane configurations for various stages of AV deployment penetration rate.*

1. Introduction

The imminent emergence of autonomous vehicles (AV) will significantly affect the future of transport systems in cities. The effects would be fundamental and comprehensive on several levels including changes on operational performance of the system and opportunities for novel congestion management strategies [1, 2]. This paper aims at understanding the fundamental characteristics of traffic flow in urban transport networks with varying levels of AV automation and realistic representation of traffic dynamics. Specifically, the methodology targets mixed traffic flow where AV and conventional human-driven vehicles interact and share the road. These mixed traffic situations correspond to eventual transition to 100% deployment of AV in long term and represent potential situations that require further investigation in the next decades.

To exploit the full potential of autonomy-enabled transport systems, this paper investigates the capacity and delay models at the link level with mixed traffic flow of AV and human-driven cars. This is crucial to lay the foundation for AV applications such as cooperative adaptive cruise control and lane-dedication schemes [3, 4]. In a mixed traffic stream, the benefit of short headway would only occur when an AV follows another AV. Therefore, in a hypothetical one-lane link scenario, the best configuration of mixed traffic would be to have two separate platoons; one for regular cars and one for AV. On the other hand, in multi-lane mixed traffic scenarios, one approach is to provide reserved lanes for AV. The performance of this strategy depends on the penetration rate and operational policies, e.g. mandatory or optional use of dedicated lanes by AV [5, 6].

This paper studies the headway and the capacity of a mixed traffic, consisting of normal and autonomous vehicles, by using a theoretical and analytical approach. Three delay profiles are derived addressing the expected, best and worst scenarios. Subsequently, the results are verified with microsimulation experiments. In addition, the paper estimates the delay of a two-lane link based on the shockwave theory. We consider four scenarios (i) two fully dedicated lanes, (ii) two fully mixed lanes, (iii) one dedicated lane for normal cars and one mixed lane, and (iv) one dedicated lane for AV and one mixed lane. The results provide analytical delay performance of every lane-dedication scenario for different penetration rates of AV. The analytical approach is based on integrating fundamental diagrams associated with flows of AV, normal cars, and mixed vehicles [7, 8].

The remainder of the paper is organized as follows. Section 2 describes the analysis of headway estimation in mixed traffic links. The expected headway and the upper and lower bounds of the headway are theoretically estimated and validated with microsimulation experiments. Section 3 introduces the delay analysis of the mixed flow in a signalized 2-lane link while Section 4 demonstrates the link delay with different AV penetration rate. Finally, Section 5 concludes the paper.

2. Headway Analysis in Mixed Traffic Links

This section investigates the properties of headway of a mixed traffic flow. The proposed modelling approach considers a platoon of vehicles that consists of (normal) human-driven and autonomous vehicles, denoted hereafter by N and AV, respectively. Furthermore, the model assumes a given penetration rate of AV, i.e. $0 \leq p \leq 1$, in the link. Intuitively, the expected headway of a mixed platoon depends on the relative locations of AV in the platoon. The model assumes distinct headway values between each two vehicles following each other. There are four different combinations, i.e. $h_{\mathrm{N-N}}, h_{\mathrm{AV-AV}}, h_{\mathrm{N-AV}}$, and $h_{\mathrm{AV-N}}$. For instance, $h_{\mathrm{AV-N}}$ denotes the expected headway $[\mathrm{s}]$ between N as the follower and AV as the leader. The value of expected headway between each two-vehicle group depends on their deployed technology. Two AV can follow each other with smaller headway because a cooperative adaptive cruise control (CACC) governs their longitudinal movements whereas in a N-AV or a AV-N situation CACC is not applicable. Additionally, with a more accurate detection sensor and a faster decision making module in an AV, we expect a lower headway between N-AV than AV-N. The headway between two normal vehicles, i.e. $h_{\mathrm{N-N}}$, is expected to be the highest headway or to be equal to $h_{\mathrm{AV-N}}$.

In the following, the best case scenario corresponding to the smallest headway is introduced. Then the worst case scenario corresponding to the largest headway is presented. Finally, the average expected headway case is investigated.

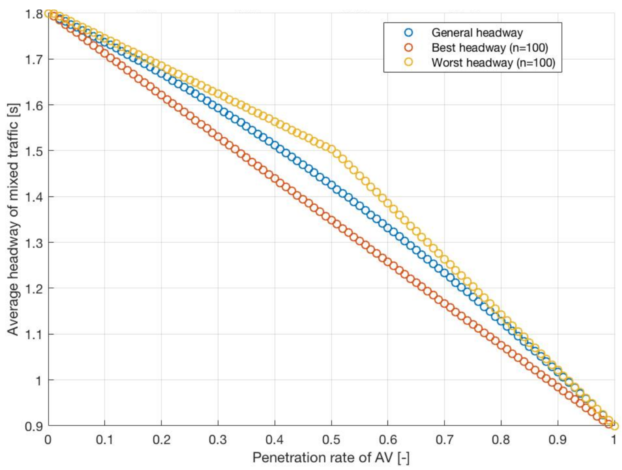

Figure 1. The lower and upper bounds and the expected value of headway of a mixed traffic. $h_{\mathrm{N-N}} = 1.8[\mathrm{s}], h_{\mathrm{AV-AV}} = 0.9[\mathrm{s}], h_{\mathrm{AV-N}} = 1.8[\mathrm{s}]$, **and** $h_{\mathrm{N-AV}} = 1.2[\mathrm{s}]$**.**

2.1. The lower bound of headway

The mixed platoon configuration that results in the smallest headway is independent of the penetration rate, p. The platoon with minimum headway consists of a group of successive human-driven vehicles and a group of consecutive AV. This is because $h_{\mathrm{AV-AV}}$ is the minimum headway among all other combinations. Let us assume a platoon of n vehicles where the expected number of AV is $n_{\mathrm{AV}} = p \cdot n$ and the expected number of N vehicles is $n_{\mathrm{N}} = (1-p) \cdot n$. Therefore, there are $n_{\mathrm{N}} - 1$ N-N combinations, $n_{\mathrm{AV}} - 1$ AV-AV combinations and 1 N-AV combination. The smallest headway $[\mathrm{s}]$ then reads:

$$\bar{\hat{h}} = \frac{(n_{\mathrm{N}} - 1) \cdot h_{\mathrm{N-N}} + (n_{\mathrm{AV}} - 1) \cdot h_{\mathrm{AV-AV}} + h_{\mathrm{N-AV}}}{n - 1}. \quad (1)$$

The minimum headway for different penetration rate of AV is depicted in Fig. 1. It is expected that with $p = 0$ the lower bound of headway approaches to $h_{\mathrm{N-N}}$ and with $p = 1$ the lower bound of headway approaches to $h_{\mathrm{AV-AV}}$.

2.2. The upper bound of headway

The mixed platoon configuration that results in the highest headway is that the every other vehicle is an AV. That is the platoon consists of groups of AV-N in which a normal car follows an AV. This can be categorized to three situations, where (i) $p < 0.5$, (ii) $p = 0.5$, and (iii) $p > 0.5$. In case of $p < 0.5$, there are more N vehicles in the stream of vehicles hence the platoon composed of groups of AV-N and a group of N vehicles following each other. Whereas with $p > 0.5$ the platoon consists of more AV than N vehicles so the platoon consists of groups of AV-N followed by a stream of AV. Accordingly, the upper bound of headway $[\mathrm{s}]$ reads as:

$$\bar{\check{h}} = \begin{cases} \frac{n_{\mathrm{AV}} \cdot h_{\mathrm{AV-N}} + (n_{\mathrm{AV}} - 1) \cdot h_{\mathrm{N-AV}} + (n_{\mathrm{N}} - n_{\mathrm{AV}}) \cdot h_{\mathrm{N-N}}}{n-1} & \text{if } p < 0.5 \\ \frac{n/2 \cdot h_{\mathrm{AV-N}} + (n/2 - 1) \cdot h_{\mathrm{N-AV}}}{n-1} & \text{if } p = 0.5 \\ \frac{n_{\mathrm{N}} \cdot h_{\mathrm{AV-N}} + n_{\mathrm{N}} \cdot h_{\mathrm{N-AV}} + (n_{\mathrm{AV}} - n_{\mathrm{N}} - 1) \cdot h_{\mathrm{AV-AV}}}{n-1} & \text{if } p > 0.5 \end{cases} \quad (2)$$

The maximum headway for different penetration rate of AV is depicted in Fig. 1. It is expected that with $p = 0$ the upper bound of headway approaches to $h_{\mathrm{N-N}}$ and with $p = 1$ the upper bound of headway approaches to $h_{\mathrm{AV-AV}}$.

2.3. The expected value of headway in a mixed platoon

Let us assume a platoon of n mixed vehicles where the AV penetration rate is p. The average headway of the platoon, $\bar{h}$ [s], is:

$$\bar{h} = \sum_{k=0}^{n} \bar{h}_k \cdot \mathcal{P}(X = k) \tag{3}$$

where k denotes the number of AV in the platoon, $\bar{h}_k$ is the expected headway in a mixed traffic flow with k AV and $n - k$ normal vehicle, and $\mathcal{P}(X = k)$ is the probability of k AV in the platoon with total n vehicles. Thus, by assuming a binomial distribution

$$\mathcal{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}. \tag{4}$$

Note that by assuming a AV penetration rate in the platoon, the number of AVs is possible to be any value between 0 and n.

Equation 3 can be rearranged in the matrix form as:

$$\bar{h} = \bar{\mathbf{H}}^T \cdot \mathbf{P} \tag{5}$$

where $\mathbf{P}$ is the vector that concatenates the probability of 0 to n AV in the platoon:

$$\mathbf{P} = \begin{bmatrix} \mathcal{P}(X=0) \\ \mathcal{P}(X=1) \\ \vdots \\ \mathcal{P}(X=n-1) \\ \mathcal{P}(X=n) \end{bmatrix} = \begin{bmatrix} \binom{n}{0} p^0 (1-p)^n \\ \binom{n}{1} p^1 (1-p)^{n-1} \\ \vdots \\ \binom{n}{n-1} p^{n-1} (1-p)^1 \\ \binom{n}{n} p^n (1-p)^0 \end{bmatrix} \tag{6}$$

and $\bar{\mathbf{H}}$ is the vector grouping the headway of a mixed traffic flow with 0 to n AV in the platoon:

$$\bar{\mathbf{H}}_{(n+1)\times 1} = \begin{bmatrix} \bar{h}_0 \\ \bar{h}_1 \\ \vdots \\ \bar{h}_{n-1} \\ \bar{h}_n \end{bmatrix} = (\mathbf{A}_{(n+1)\times 4} \cdot \mathbf{H}_{4\times 1} \times \frac{1}{n-1}) \circ \mathbf{C}_{(n+1)\times 1}. \tag{7}$$

The vector $\mathbf{H}$ and $\mathbf{C}$ are as following:

$$\mathbf{H} = \begin{bmatrix} h_{\mathrm{N-N}} \\ h_{\mathrm{AV-AV}} \\ h_{\mathrm{AV-N}} \\ h_{\mathrm{N-AV}} \end{bmatrix} \tag{8}$$

and

$$\mathbf{C} = \begin{bmatrix} \frac{1}{\binom{n}{0}} \\ \frac{1}{\binom{n}{1}} \\ \vdots \\ \frac{1}{\binom{n}{n-1}} \\ \frac{1}{\binom{n}{n}} \end{bmatrix}. \quad (9)$$

In addition, $\circ$ denotes the Hadamard (element-wise) product and matrix $\mathbf{A}$ counts the number of each combination of N-N, AV-AV, AV-N, and N-AV assuming k AV and $n-k$ normal vehicle and a binomial distribution of AV in the platoon. The derivation of $\mathbf{A}$ is straightforward however cumbersome and due to the space limitation is not provide here. However, Equation 5 can be constructed readily given p and $\mathbf{H}$.

Fig. 1 shows the expected headway of a mixed traffic flow with different AV penetration rates. The headway values are assumed according to [3, 4]. As expected, with $p = 0$ the expected headway is equal to $h_{\mathrm{N-N}}$ and with $p = 1$ the expected headway is equal to $h_{\mathrm{AV-AV}}$. It is worth to note that, the expected value of headway is independent of the number of cars in the platoon, i.e. n, and is therefore only a function of the penetration rate, p, and the values used for the headways. Whereas the upper and lower bound of headway depend on the platoon size n as well.

Ultimately, we test extensive scenarios in Aimsun microsimulation environment where the driving behaviour of AV and normal vehicles are modelled. This is done by modifying the car-following characteristics of AV and normal vehicles. The tests (10 randomly selected replications) include the headway measurement of mixed traffic flows with different penetration rates of AV. In addition, the worst and best case scenarios are also replicated in Aimsun. Fig. 2 shows the results of average headway, upper, and lower bounds of headways for different penetration rates of AV in a mixed flow. Fig. 2 demonstrates that the theoretical derivation of headways are accurately consistent with the measured headway from microsimulation experiments.

3. Delay Analysis in Mixed Traffic

This section derives the queueing delay of a 2-lane signalized link with mixed AV and normal vehicles. The delay estimation method is based on shockwave theory and assumes a triangle fundamental diagram (FD). The characteristics of FD, i.e. capacity, critical density and jam density, for each type of flows, i.e. mixed, only AV, and only normal vehicles are different. We consider four types of lane-configurations, (i) 2 dedicated lanes, (ii) 2 mixed lanes, (iii) 1 mixed lane and 1 dedicated lane for normal vehicles, and (iv) 1 mixed lane and 1 dedicated lane for AV.

Note that for the sake of simplicity we consider a link with 2 lanes. The model can be extended to links with several lanes. The calculation would follow similar approach with more extensive considerations.

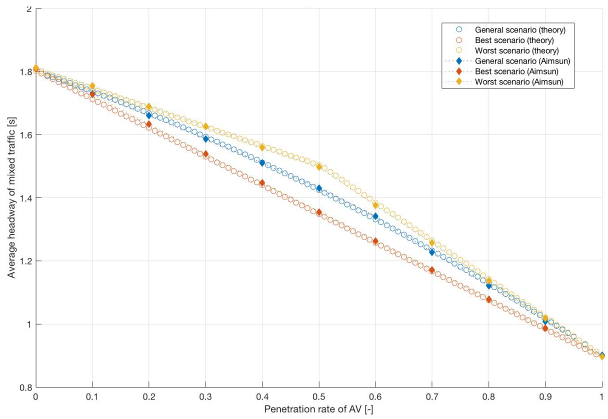

Figure 2. The minimum, maximum and the expected value of headway in a mixed traffic for different penetration rates of AV. The filled diamonds depict the results of the microsimulation experiments. The circles are the theoretical headway. $h_{\mathrm{N-N}} = 1.8[\mathrm{s}], h_{\mathrm{AV-AV}} = 0.9[\mathrm{s}], h_{\mathrm{AV-N}} = 1.8[\mathrm{s}]$, **and** $h_{\mathrm{N-AV}} = 1.2[\mathrm{s}]$**.**

3.1. Delay in 2 dedicated lanes

Consider a triangle FD for each type of vehicles (i.e. AV and normal vehicles) where free flow speed $u\,[\mathrm{m/s}]$, jam density $k^{\mathrm{j}}\,[\mathrm{veh/m}]$, shockwave speed, $w\,[\mathrm{m/s}]$, and capacity $c\,[\mathrm{veh/s}]$ are different for each type. The delay of single lane i reads as:

$$D_i = 0.5R_i^2 \cdot (\frac{w_i \cdot u_i}{w_i - u_i}) \tag{10}$$

where $D_i\,[\mathrm{m.s}]$ is link i delay during a cycle with red duration R_i. Note that D_i can be readily converted to the $[\mathrm{veh.s}]$ unit by multiplying with $k^{\mathrm{j}}\,[\mathrm{veh/m}]$. Equation 10 can be rewritten based on capacity, jam density, arrival flow, and free flow speed. Consequently, the total delay for both dedicated lanes is:

$$D_{\mathrm{T}} = \sum_{i=1}^{2} 0.5R_i^2 \cdot (\frac{\frac{c_i}{k_i^{\mathrm{j}} - \frac{c_i}{u_i^{\mathrm{f}}}} \cdot \frac{q_i}{k_i^{\mathrm{j}} - \frac{q_i}{u_i^{\mathrm{f}}}}}{\frac{c_i}{k_i^{\mathrm{j}} - \frac{c_i}{u_i^{\mathrm{f}}}} - \frac{q_i}{k_i^{\mathrm{j}} - \frac{q_i}{u_i^{\mathrm{f}}}}}) \tag{11}$$

where $q_i\,[\mathrm{veh/s}]$ is the arrival flow of lane i. Hence, the arrival flow to the lane dedicated to normal vehicles is $q_1 = (1-p) \cdot q_{\mathrm{T}}$ and the arrival flow to the lane dedicated to AV is $q_2 = p \cdot q_{\mathrm{T}}$. $q_{\mathrm{T}}\,[\mathrm{veh/s}]$ is the total arrival flow to the link.

3.2. Delay in 2 mixed lanes

Based on the proposed headway model in Section 2, the capacity of a mixed traffic flow is reciprocal of the expected headway, i.e. $c_{\mathrm{m}} = 1/\bar{h}$. Hence $c_{\mathrm{m}}\,[\mathrm{veh/s}]$ depends on the AV penetration rate p and the values of the headways $h_{\mathrm{N-N}} = 1/c_1, h_{\mathrm{AV-AV}} = 1/c_2$, $h_{\mathrm{N-AV}}$, and $h_{\mathrm{AV-N}}$. Note that c_1 and c_2 are the capacities of normal vehicles and AV streams respectively.

To establish the delay relationship, we assume that the mixed traffic stream has similar free flow speed ($u_{\mathrm{m}}^{\mathrm{f}}$) and jam density ($k_{\mathrm{m}}^{\mathrm{j}}$) to the two types of vehicles (AV and normal). In addition, it is assumed that the delay of both lanes are equal. Consequently, the total delay of 2 lanes is:

$$D_{\mathrm{T}} = 2 \cdot 0.5 R_{\mathrm{m}}^2 \cdot \left(\frac{\frac{c_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{c_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}} \cdot \frac{q_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{q_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}}}{\frac{c_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{c_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}} - \frac{q_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{q_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}}} \right) \tag{12}$$

where $q_{\mathrm{m}} = q_{\mathrm{T}}/2$, q_{T} [veh/s] is the total arrival flow to the link, and R_{m} [s] is the red interval.

3.3. Delay in 1 dedicated lane for normal vehicles and 1 mixed lane

The total delay of this lane configuration can be derived based on (11) and (12).

$$D_{\mathrm{T}} = 0.5 R_{\mathrm{m}}^2 \cdot \left(\frac{\frac{c_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{c_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}} \cdot \frac{q_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{q_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}}}{\frac{c_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{c_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}} - \frac{q_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{q_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}}} \right) + 0.5 R_1^2 \cdot \left(\frac{\frac{c_1}{k_1^{\mathrm{j}} - \frac{c_1}{u_1^{\mathrm{f}}}} \cdot \frac{q_1}{k_1^{\mathrm{j}} - \frac{q_1}{u_1^{\mathrm{f}}}}}{\frac{c_1}{k_1^{\mathrm{j}} - \frac{c_1}{u_1^{\mathrm{f}}}} - \frac{q_1}{k_1^{\mathrm{j}} - \frac{q_1}{u_1^{\mathrm{f}}}}} \right) \tag{13}$$

where q_1, c_1, k_1^{j}, and u_1^{f} are respectively the arrival flow, capacity, jam density, and free flow speed of the dedicated lane for normal vehicles; q_{m}, c_{m}, $k_{\mathrm{m}}^{\mathrm{j}}$, and $u_{\mathrm{m}}^{\mathrm{f}}$ are respectively the arrival flow, capacity, jam density, and free flow speed of the mixed lane. R_1 and R_{m} are the red phase duration for the dedicated and mixed lanes respectively. Note that, there is a distinction between the red phases of each lane because the optimal isolated traffic signal timing plan with the introduction of AV might be different for AV and normal vehicles. This is because a fixed number of AV need less green time to discharge compare to the similar number of normal vehicles as AV have lower headway and less reaction time (i.e. lower lost time).

A key modelling point is to determine the allocation of the total inflow to the link between the mixed and dedicated lanes (i.e. q_{m} and q_1). Evidently, a part (α_1) of normal vehicle demand select the dedicated lane while the other part of the normal vehicle demand and the AV demand choose the mixed lane.

$$q_1 = \alpha_1 \cdot (1-p) \cdot q_{\mathrm{T}} \tag{14}$$

$$q_{\mathrm{m}} = (1-\alpha_1) \cdot (1-p) \cdot q_{\mathrm{T}} + p \cdot q_{\mathrm{T}}. \tag{15}$$

We assume that the normal vehicles flow allocation is such that in a long run a user equilibrium condition governs the lane choice of normal vehicles. That is the ratio of arrival rate and exit rate for both lanes would be equal, $q_{\mathrm{m}}/c_{\mathrm{m}} = q_1/c_1$. On the other hand, the capacity of the mixed lane, c_{m}, is a function $f(.)$ of AV penetration rate in lane 2, p_2.

$$c_{\mathrm{m}} = f(p_2) = f\left(\frac{p}{p + (1-\alpha_1) \cdot (1-p)}\right). \tag{16}$$

Therefore to estimate α_1, we approximate the mixed traffic headway as a function of the AV penetration rate as, $1/c_{\mathrm{m}} = 0.3p_2^2 - 0.6p_2 + 1.8$. This is the best (minimum distance) 2nd order curve fit to the blue curve in Fig. 2. Readily the value of α_1 can be estimated based on the AV penetration rate.

3.4. Delay in 1 dedicated lane for AV and 1 mixed lane

Finally, the last lane configuration is to consider a dedicated lane for AV and a mixed lane. The total delay is estimated similar to the approach introduced in subsection 3.3.

$$D_{\mathrm{T}} = 0.5R_{\mathrm{m}}^2 \cdot (\frac{\frac{c_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{c_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}} \cdot \frac{q_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{q_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}}}{\frac{c_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{c_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}} - \frac{q_{\mathrm{m}}}{k_{\mathrm{m}}^{\mathrm{j}} - \frac{q_{\mathrm{m}}}{u_{\mathrm{m}}^{\mathrm{f}}}}}) + 0.5R_2^2 \cdot (\frac{\frac{c_2}{k_2^{\mathrm{j}} - \frac{c_2}{u_2^{\mathrm{f}}}} \cdot \frac{q_2}{k_2^{\mathrm{j}} - \frac{q_2}{u_2^{\mathrm{f}}}}}{\frac{c_2}{k_2^{\mathrm{j}} - \frac{c_2}{u_2^{\mathrm{f}}}} - \frac{q_2}{k_2^{\mathrm{j}} - \frac{q_2}{u_2^{\mathrm{f}}}}}) \tag{17}$$

where q_2, c_2, k_2^{j}, and u_2^{f} are respectively the arrival flow, capacity, jam density, and free flow speed of the dedicated lane for AV; q_{m}, c_{m}, $k_{\mathrm{m}}^{\mathrm{j}}$, and $u_{\mathrm{m}}^{\mathrm{f}}$ are respectively the arrival flow, capacity, jam density, and free flow speed of the mixed lane. R_2 and R_{m} are the red phase duration for the dedicated and mixed lanes respectively. Note that, there is a distinction between the red phases of each lane because the optimal isolated traffic signal timing plan with the introduction of AV might be different for AV and mixed flows.

Similarly, a key modelling point is to determine the allocation of the total AV inflow to the link between the mixed and dedicated lanes (i.e. q_{m} and q_2), where a part (α_2) of the AV demand select the dedicated lane while the other part of the AV demand and the normal vehicle demand choose the mixed lane.

$$q_2 = \alpha_2 \cdot p \cdot q_{\mathrm{T}} \tag{18}$$
$$q_{\mathrm{m}} = (1-p) \cdot q_{\mathrm{T}} + (1-\alpha 2) \cdot p \cdot q_{\mathrm{T}}. \tag{19}$$

We assume that the AV flow allocation is such that in a long run a user equilibrium condition governs the lane choice of AV. That is, the ratio of arrival rate and exit rate for both lanes would be equal, $q_{\mathrm{m}}/c_{\mathrm{m}} = q_2/c_2$. In addition, the capacity of the mixed lane, c_{m}, is a function $f(.)$ of AV penetration rate.

$$c_{\mathrm{m}} = f(\frac{(1-\alpha 2) \cdot p}{(1-\alpha 2) \cdot p + (1-p)}). \tag{20}$$

Therefore to estimate α_2, we use the best 2nd order fit for the mixed traffic headway as a function of the AV penetration rate, $1/c_{\mathrm{m}} = 0.3p^2 - 0.6p + 1.8$. The value of α_2 can be readily estimated based on the AV penetration rate.

4. Numerical tests

The effect of AV penetration rate on the delay performance of the four lane configurations is studied. The parameters are set as: $c_1 = 1/1.8$[veh/s], $c_2 = 1/0.9$[veh/s], $u_1^{\mathrm{f}} = u_2^{\mathrm{f}} = u_{\mathrm{m}}^{\mathrm{f}} = 50\,[\mathrm{km/h}]$, $k_1^{\mathrm{j}} = k_2^{\mathrm{j}} = k_{\mathrm{m}}^{\mathrm{j}} = 150[\mathrm{km/h}]$, $R_1 = R_2 = R_{\mathrm{m}} = 30\,[\mathrm{s}]$, $G_1 = G_2 = G_{\mathrm{m}} = 30\,[\mathrm{s}]$, $h_{\mathrm{N-N}} = 1.8\,[\mathrm{s}], h_{\mathrm{AV-AV}} = 0.9\,[\mathrm{s}], h_{\mathrm{AV-N}} = 1.8\,[\mathrm{s}]$, and $h_{\mathrm{N-AV}} = 1.2\,[\mathrm{s}]$.

Figure 3 shows the total delay of each lane configuration for different AV penetration rates. Intuitively, the delays of dedicated lanes and 1 AV dedicated lane and 1 mixed lane are similar with penetration rate of AV equal to zero. Additionally, with penetration rate of AV equal to one (i.e. no normal vehicle in the traffic stream) the delays of dedicated lanes and 1 lane dedicated to normal vehicles and 1 mixed lane are similar. It is worth to

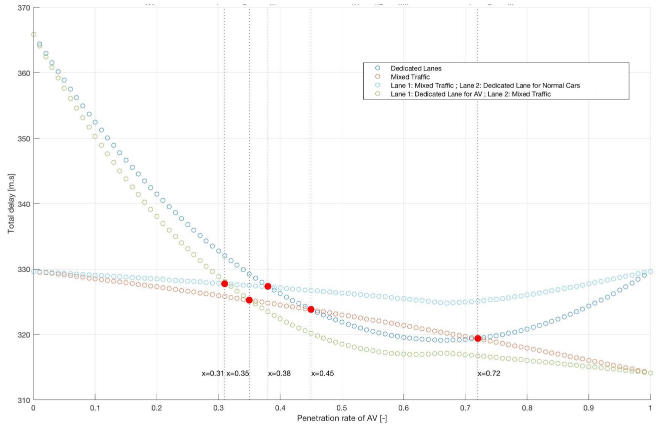

Figure 3. The total delay of each lane configuration for different AV penetration rates in a mixed traffic.

point out that the link configuration with minimum delay depends on the penetration rate of AV. For penetration rates up to 0.35 the mixed lanes have the minimum delay. With the increase in the percentage of AV in the traffic flow the best configuration is to reserve a lane for AV and a mixed lane. This is of great importance for future deployment of AV in cities.

5. Summary and Future Work

The effects of introduction of autonomous vehicles (AV) in a mixed traffic flow with conventional human-driven vehicles has been explored in this paper. It is assumed that the AV share the infrastructure with normal vehicles. The proposed model specifically derives analytical relationships for the headway, capacity, and delay of a mixed traffic flow with different AV penetration rates. The headway of a mixed traffic is a stochastic variable that is a function of exact locations of AV in the platoon. The expected value of headway is formulated based on the penetration rate of AV, and the headways between AV and AV, AV and normal car, normal car and AV, and normal car and normal car. Furthermore, the lower and upper bounds of the headway are derived.

Extensive microsimulation tests have been studied where the driving behaviour and car-following dynamics of vehicles are modified to replicate the mixed traffic flow of AV and normal vehicles. The microsimulation headway measurements and analytical relationships are shown to be consistent. The analytical headways are then integrated with shockwave theory to estimate delay of a 2-lane signalized link. The delay of four lane configurations, i.e. 2 mixed lanes, 2 dedicated lanes, and 1 mixed lane and 1 dedicated lane for AV and normal vehicles are estimated. The delay performance of each lane configuration for different AV penetration rates are compared and the optimum is indicated.

This work is fundamental for devising a holistic traffic flow modelling and control in mixed AV and normal vehicles environment. Future research can design traffic signal settings for the mixed traffic flow based on the proposed model to increase the intersection throughput [9] and reduce the probability of spillback in arterials [10] with optimally utilizing the AV headway characteristics.

References

[1] Bu, F., Tan, H.S. and Huang, J., 2010, June. Design and field testing of a cooperative adaptive cruise control system. In American Control Conference (ACC), 2010 (pp. 4616-4621). IEEE.

[2] Christofa, E., Argote, J. and Skabardonis, A., 2013. Arterial queue spillback detection and signal control based on connected vehicle technology. Transportation Research Record: Journal of the Transportation Research Board, (2356), pp.61-70.

[3] Shladover, S., Su, D. and Lu, X.Y., 2012. Impacts of cooperative adaptive cruise control on freeway traffic flow. Transportation Research Record: Journal of the Transportation Research Board, (2324), pp.63-70.

[4] Vander Werf, J., Shladover, S., Miller, M. and Kourjanskaia, N., 2002. Effects of adaptive cruise control systems on highway traffic flow capacity. Transportation Research Record: Journal of the Transportation Research Board, (1800), pp.78-84.

[5] Talebpour, A. and Mahmassani, H.S., 2016. Influence of connected and autonomous vehicles on traffic flow stability and throughput. Transportation Research Part C: Emerging Technologies, 71, pp.143-163.

[6] Wang, M., Hoogendoorn, S.P., Daamen, W., van Arem, B., Shyrokau, B. and Happee, R., 2016. Delay-compensating strategy to enhance string stability of adaptive cruise controlled vehicles. Transportmetrica B: Transport Dynamics, pp.1-19.

[7] Coifman, B., 2014. Revisiting the empirical fundamental relationship. Transportation Research Part B: Methodological, 68, pp.173-184.

[8] Tanaka, K., Nagatani, T. and Masukura, S., 2008. Fundamental diagram in traffic flow of mixed vehicles on multi-lane highway. Physica A: Statistical Mechanics and its Applications, 387(22), pp.5583-5596.

[9] Yang, K., Guler, S.I. and Menendez, M., 2016. Isolated intersection control for various levels of vehicle technology: Conventional, connected, and automated vehicles. Transportation Research Part C: Emerging Technologies, 72, pp.109-129.

[10] Ramezani, M., de Lamberterie, N., Skabardonis, A. and Geroliminis, N., 2017. A link partitioning approach for real-time control of queue spillbacks on congested arterials. Transportmetrica B: Transport Dynamics, 5(2), pp.181-194.