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A robust signal setting for urban road network with hazard materials transportation[1]

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Abstract

For urban road network with hazard materials transportation, it is of paramount importance for decision makers to determine a robust signal setting against a high-consequence of imminent exposure risk in the presence of hazmat transportation. In order to simultaneously minimize total travel delay over entire road network and mitigate uncertain risk, a scenario-based bi-level programming problem (SBLPP) is proposed. Due to non-linearity of constraints, a new solution method is presented. To demonstrate effectiveness and feasibility of proposed model, numerical computations using example road network are empirically made. These results indicate that proposed model can achieve reliably better results than did those recently proposed with reasonable sub-optimality in deterministic conditions.

1. Introduction

Because of imminent exposure risk associated with the accidental release of hazard materials (hazmat), the people living and working around the roads heavily used for hazmat transportation incur most of the risk during transportation (List et al. 2003; Erkut and Verter 1995). For most urban road networks with signal-controlled junctions, severe travel delay and transportation risk would be incurred by all road users as a result of insufficient provision of link capacity in the presence of uncertainty (Yin 2008; Ukkusuri et al. 2010; Tong et al. 2015; Liu et al. 2015). A hazmat transportation network design problem (HNDP) has long been deemed one most challenging issue facing transportation policy decision makers (Kara and Verter 2004; Bianco et al. 2009; Gzara 2013; Chiou 2016). Among which the bi-level programming model (Kara and Verter 2004) has been widely recognized a good alternative to formulate a leader-follower relationship between different decision makers at various level in the presence of different prospective. In order to determine a resilient signal setting that is feasible for any realization of uncertain travel demand, Yin (2008) proposed a set-based robust signal setting (SR) model. Chiou (2016) presented a deterministic signal setting (NR) to minimize total travel cost. In this paper, a flexible signal setting (FS) is presented to minimize travel delay and mitigate stochastic risk over entire road network, and a scenario-based bi-level programming problem (SBLPP) is proposed. Due to non-linearity of SBLPP, a new solution method is presented. To demonstrate effectiveness of proposed model, numerical computation and comparison are made with recently proposed. These results reported obviously indicate that proposed FS can achieve reliably better results than SR and NR.

2. A user equilibrium assignment for HNDP

According to Wardrop's first principle, a user equilibrium assignment can be formulated as a variational inequality with signal delay incurred by users at downstream junction. Notation used throughout this paper is presented first.

2.1. Notation

Let $G(N,L)$ denote a directed road network, where *N* represents a set of signal controlled junctions and *L* represents a set of links denoted by *a*, $\forall a \in L$. Each traffic stream approaching any junction is represented by its own link.

W , W^H - a set of origin-destination (OD) pairs respectively for regular traffic and hazmat transportation.

R_i - a set of routes between OD pair *i.*

$Q=[Q_i]$, $q=[q_i]$ - matrix of OD pairs respectively for regular and hazmat traffic demands.

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[1] "This is an abridged version of the paper presented at the conference. The full version is being submitted elsewhere. Details on the full paper can be obtained from the author."

ζ - the reciprocal of common cycle time.

$\zeta_{\min}, \zeta_{\max}$ - the minimum and maximum reciprocal of the common cycle time.

$\theta = [\theta_{jm}]$ - the vector of start of green for various links as proportions of cycle time where θ_{jm} is start of next green for signal group *j* at junction *m*.

$\phi = [\phi_{jm}]$ - the vector of duration of green for various links as proportions of cycle time where ϕ_{jm} is the duration of green for signal group *j* at junction *m*.

τ_{ijm} - the clearance time between the end of green for signal group *i* and the start of green for incompatible signal group *j* at junction *m*.

$\Psi = (\zeta, \theta, \phi)$ - the set of signal setting variables respectively for the reciprocal of common cycle time, start and duration of greens.

g_a , $g_{\min}$ - duration of effective green and minimum green for link *a*.

$\Omega_m(i, j)$ - collection of numbers 0 and 1 for each pair of incompatible signal groups at junction *m*, where $\Omega_m(i, j) = 0$ if the start of green for signal group *i* precedes that of j and 1, otherwise.

D_a , S_a - the rate of delay and the number of stops on link *a*.

W_D , W_S - weighting factor for rate of delay and number of stops.

M_D , M_S - monetary factor associated with rate of delay and number of stops.

ρ_a - maximum degree of saturation for link *a*.

s_a - saturation flow on link *a*.

$\bar{r} = [\bar{r}_a]$ - the vector of nominal risk of accidental release of hazmat on link ***a*.**

$\hat{r} = [\hat{r}_a]$ - the vector of step-length risk of accidental release of hazmat on link ***a*.**

$r^s = [r_a^s]$ - the vector of scenario-based risk of accidental release of hazmat on link *a* such that $r_a^s \in [\bar{r}_a , \bar{r}_a + \hat{r}_a]$ **.**

p_a^s - a scenario-based incidental probability of accidental release of hazmat on link *a*.

Γ - budget of all uncertain link risk r^s in all scenarios $\forall s \in L$.

f_a , x_a - link flow respectively for regular and hazmat traffic.

h_k - traffic flow on path k between OD trips.

λ - link-path incidence matrix with entry $\lambda_{ak} = 1$ if path *k* uses link *a*, and 0 otherwise.

Λ - OD-path incidence matrix with entry $\Lambda_{wk} = 1$ if path *k* connects OD trip w , and 0 otherwise.

c_a , C_k - travel time on link *a* and path *k*.

$c_{a,0}$ - cruise travel time on link *a*.

d_a - average delay on link *a*.

σ_c - a converting factor from expected risk to travel time.

σ - a converting factor from risk to monetary factor.

2.2. A user equilibrium signal settings

For a signal-controlled road network, the link travel time can be calculated as a sum of cruise travel time, and signal delay at downstream.

$$c_a(\Psi, f_a) = c_{a,0} + d_a(\Psi, f_a) \quad , \quad \forall a \in L \qquad \text{Eq. (1)}$$

According to Wardrop's first principle, user equilibrium assignment with signal settings can be formulated as a following mathematical optimization:

$$\underset{f_a(\Psi)}{Min} \quad \sum_{a \in L} \int_0^{f_a(\Psi)} c_a(\Psi, t) dt \qquad \text{Eq. (2)}$$

$$\textit{subject to} \quad \sum_{k \in R_i} h_k = Q_i \,, \forall i \in W$$

$$f_a = \sum_{i \in W} \sum_{k \in R_i} \lambda_{ak} h_k \,, \forall a \in L$$

$$h_k \geq 0, \forall k \in \bigcup_{i \in W} R_i \,,$$

Let Π_f denote a feasible set for regular traffic flow such that

$$\Pi_f = \{ f : f = \lambda h, \Lambda h = Q, h \geq 0 \} \qquad \text{Eq. (3)}$$

User equilibrium traffic flow can be also determined by a variational inequality. For feasible flow f', $f' \in \Pi_f$, it is to find a flow $f(\Psi) \in \Pi_f$ such that

$$c(\Psi, f)(f' - \mathrm{f}(\Psi)) \geq 0 \qquad \text{Eq. (4)}$$

The solution set for Eq. (3) can be denoted by $\Omega_f(\Psi)$.

3. A randomized user equilibrium assignment

Let z_a^s denote a scaled deviation for random link risk *r* in scenario *s*, $\forall a, s \in L$. According to definition, a scenario-based link risk r_a^s lies in a polyhedron $[\bar{r}_a, \bar{r}_a + \hat{r}_a]$ within a budget Γ such that for each link *a* in scenario *s*, it can be determined by

$$r_a^s = \bar{r}_a + z_a^s \hat{r}_a \,, \quad \forall a \in L \qquad \text{Eq. (5)}$$

and

$$\sum_{a,s \in L} z_a^s \leq \Gamma \,; \; z_a^s \in \{0,1\}, \forall a, s \in L \qquad \text{Eq. (6)}$$

According to definition in Eq. (1), a link travel time for hazmat traffic with scenario-based probability p_a^s can be approximately expressed as a linear combination of link travel time function $c_a(\Psi, f_a)$ and probabilistic link risk through converting factor σ_c to travel time as follows.

$$c_a(\Psi, f_a) + \sigma_c \sum_{s \in L} p_a^s r_a^s \,, \; \forall a \in L \qquad \text{Eq. (7)}$$

A randomized user equilibrium taking account of probabilistic risk in scenarios can be presented.

$$\underset{x_a(\Psi)}{Min} \quad \sum_{a \in L} \int_0^{x_a(\Psi)} c_a(\Psi, f_a) + \sigma_c \sum_{s \in L} p_a^s r_a^s dt \qquad \text{Eq. (8)}$$

$$\textit{subject to} \quad \sum_{k \in R_i} h_k = q_i \,, \forall i \in W^H$$

$$x_a = \sum_{i \in W^H} \sum_{k \in R_i} \lambda_{ak} h_k \,, \forall a \in L$$

$$h_k \geq 0, \forall k \in \bigcup_{i \in W^H} R_i \,,$$

Again, let Π_x denote a feasible set for hazmat traffic with travel demand such that

$$\Pi_x = \{ x : x = \lambda h, \Lambda h = q, h \geq 0 \} \qquad \text{Eq. (9)}$$

Let randomized link travel time for hazmat carriers denoted by $\widehat{c}(\Psi, f)$, i.e.

$$\widehat{c}(\Psi, f) = c(\Psi, f) + \sigma_c \sum_{s \in L} p^s r^s \qquad \text{Eq. (10)}$$

A variational inequality for Eq. (8) can be presented if and only if the gradient of objective function for Eq. (8) is available, i.e. for x', $x' \in \Pi_x$, it is to find a flow $x(\Psi) \in \Pi_x$ such that

$$\widehat{c}(\Psi, f)(x' - x(\Psi)) \geq 0 \qquad \text{Eq. (11)}$$

The solution set for Eq. (11) can be denoted by $\Omega_x(\Psi)$.

4. A scenario-based bi-level programming problem (SBLPP)

A flexible signal setting (FS) with randomized set of link travel time can be formulated as a scenario-based bi-level programming problem (SBLPP).

4.1. A scenario-based signal setting

Let signal settings constraint set denoted by Π_Ψ and specified below. For cycle time constraint, it implies

$$\zeta_{\min} \leq \zeta \leq \zeta_{\max} \qquad \text{Eq. (12)}$$

For each signal controlled junction *m*, the phase *j* green time for all signal groups at junction *m* can be expressed as

$$g_{\min}\zeta \leq \phi_{jm} \leq 1, \forall j, m \qquad \text{Eq. (13)}$$

The link capacity for all links leading to junction *m* can be expressed as

$$f_a \leq \rho_a s_a g_a, \forall a \in L \qquad \text{Eq. (14)}$$

and the clearance time τ_{ijm} for incompatible signal groups *i* and *j* at junction *m* can be expressed as

$$\theta_{im} + \phi_{im} + \tau_{ijm}\zeta \leq \theta_{jm} + \Omega_m(i, j), \quad \forall i, j, m \qquad \text{Eq. (15)}$$

According to Chiou (2016), the performance index (PI) can be taken as a weighted sum of a linear combination of rate of delay, and number of stop per unit time for all links *a*, and probabilistic link risk imposed by hazmat traffic with a weighting parameter α, $0 \leq \alpha \leq 1$. Let PI denoted by P_0, it implies

$$P_0 = (1-\alpha)\left(\sum_{a \in L} D_a(\Psi, f_a) W_D M_D + S_a(\Psi, f_a) W_S M_S\right) + \alpha\sigma \sum_{a \in L} x_a \sum_{s \in L} p_a^s r_a^s \qquad \text{Eq. (16)}$$

A flexible signal setting with scenario-based risk can be determined by a scenario-based bi-level programming problem (SBLPP).

$$\underset{\Psi \in \Pi_\Psi, f \in \Omega_f, x \in \Omega_x}{Min} \; \underset{p}{Max} \; P_0(\Psi, f, x, p) \qquad \text{Eq. (17)}$$

$$\textit{subject to} \quad \sum_{s \in L} p_a^s = 1, \forall a \in L$$

$$\text{and for all} \begin{pmatrix} f' \in \Pi_f \\ x' \in \Pi_x \end{pmatrix}, \begin{pmatrix} f \\ x \end{pmatrix} \text{ solves} \begin{pmatrix} c(\Psi, f)(f' - f(\Psi)) \geq 0 \\ \widehat{c}(\Psi, f)(x' - x(\Psi)) \geq 0 \end{pmatrix}$$

5. A new solution method

The SBLPP in Eq. (17) can be solved by a single-level min-max model with optimal signal settings and probabilities (Ψ^*, p^*). Let

$$\breve{P} = \underset{\Psi \in \Pi_\Psi}{Min} P_1(\Psi, p) \qquad \text{Eq. (18)}$$

and

$$\widehat{P} = \underset{0 \le p \le 1}{Max} P_1(\Psi, p) \qquad \text{Eq. (19)}$$

To find (Ψ^*, p^*) in Eqs. (18) and (19) such that

$$p^* = Arg \underset{0 \le p \le 1}{Max} \breve{P} \qquad \text{Eq. (20)}$$

and

$$\Psi^* = Arg \underset{\Psi \in \Pi_\Psi}{Min} \widehat{P} \qquad \text{Eq. (21)}$$

when the following condition holds:

$$\breve{P} \le \breve{P}^* = \widehat{P}^* \le \widehat{P} \qquad \text{Eq. (22)}$$

5.1. Linear program for maximum

An effective upper bound in Eq. (18) can be determined by a linear maximization.

$$\widehat{P} = \underset{p}{Max} (1-\alpha)\left(\sum_{a \in L} D_a(\Psi) W_D M_D + S_a(\Psi) W_S M_S\right) + \alpha\sigma \sum_{a \in L} x_a \sum_{s \in L} p_a^s r_a^s \qquad \text{Eq. (23)}$$

subject to $\sum_{s \in L} p^s = 1$

5.2. A novel approach for minimum

An effective lower bound in Eq. (18) can be determined by a minimization.

$$\breve{P} = \underset{\Psi \in \Pi_\Psi}{Min} (1-\alpha)\left(\sum_{a \in L} D_a(\Psi) W_D M_D + S_a(\Psi) W_S M_S\right) + \alpha\sigma \sum_{a \in L} x_a \sum_{s \in L} p_a^s r_a^s \qquad \text{Eq. (24)}$$

According to Eq. (20), it implies

$$\breve{P} = \underset{\Psi \in \Pi_\Psi}{Min} \widehat{P}(\Psi) \qquad \text{Eq. (25)}$$

Moreover,

$$\partial \widehat{P} = co\left\{ \lim_{k \to \infty} \nabla \widehat{P}^k : \Psi^k \to \Psi^*, \nabla \widehat{P}^k exists \right\} \qquad \text{Eq. (26)}$$

For PI in Eq. (24), a linear approximation at signal setting Ψ^k along a perturbed direction $\Delta\Psi^k$ can be established using a bundle of gradients $\left\{\nabla \widehat{P}^i = \nabla \widehat{P}(\Psi^i); 1 \le i \le k\right\}$. Let $\overline{P}^k$ denote a linear approximation of $\widehat{P}^k$ close to Ψ^k at iteration *i*, $1 \le i \le k$, it implies

$$\overline{P}^k = \underset{1 \le i \le k}{Max} \left\{ \nabla \widehat{P}^i (\Psi - \Psi^i) + \widehat{P}^i \right\} \qquad \text{Eq. (27)}$$

Let

$$e_{i,k} = \widehat{P}^k - (\widehat{P}^i + \nabla \widehat{P}^i (\Psi^k - \Psi^i)) \qquad \text{Eq. (28)}$$

denote an error bound for a linear approximation of $\widehat{P}^k$. A linear approximation of $\widehat{P}^k$ can be expressed in terms of gradients.

$$\bar{P}^k = \underset{1 \le i \le k}{Max} \left\{ \nabla \hat{P}^i (\Psi - \Psi^k) - e_{i,k} \right\} + \hat{P}^k \quad \text{Eq. (29)}$$

Therefore, a cutting plane for Eq. (24) can be expressed as follows.

$$\underset{\Psi \in \Pi_\Psi}{Min} \bar{P}^k = \underset{\Psi \in \Pi_\Psi}{Min} \underset{1 \le i \le k}{Max} \left\{ \nabla \hat{P}^i (\Psi - \Psi^k) - e_{i,k} \right\} + \hat{P}^k \quad \text{Eq. (30)}$$

Let $\breve{\Psi}^k$ solve cutting plane in Eq. (30), and $\Pr_{\Pi_\Psi}(\Psi^k)$ denote the projection of Ψ^k on constraint set Π_Ψ such that

$$\left\| \Psi^k - \Pr_{\Pi_\Psi}(\Psi^k) \right\| = \inf_{z \in \Pi_\Psi} \left\| \Psi^k - z \right\| \quad \text{Eq. (31)}$$

A sequence of iterates $\{\Psi^k\}$ can be determined in accordance with

$$\Psi^{k+1} = \Pr_{\Pi_\Psi}(\Psi^k + \tau(\breve{\Psi}^k - \Psi^k)), \quad \forall k = 1,2,... \quad \text{Eq. (32)}$$

where $\tau \in (0,2)$ is the step length which minimizes $\bar{P}^k$ in Eq. (30).

5.3. A solution scheme

A flexible signal setting (FS) with a randomized set of link risk can be determined by the following steps.

Step 1. Start with initial signal setting Ψ^k and randomized probability p^k of uncertain risk on links. Set iteration index $k = 1$ and a stopping threshold ε^k.

Step 2. Solve a linear program for maximum $\hat{P}^k$ and randomized probability p^k.

Step 3. Solve a minimum $\breve{P}^k$ and a tentative signal setting $\breve{\Psi}^k$ by Eq. (30).

Step 4. Compute new signal setting Ψ^{k+1} by Eq. (32) and calculate bound gap between maximum and minimum.

Step 5. Bound check: if the bound gap δ^k is within a threshold ε^k, then stop and Ψ^{k+1} is the solution. Otherwise, move iteration k to $k+1$ and go to Step 2.

6. Numerical computation and comparison

Numerical computations using a real-data Sioux Falls city network (Suwansirikul et al. 1987) with 6 signal-controlled junctions are performed for (17), as seen in Fig. 1. The minimum green time for each signal-controlled group is 7 sec using typical values found in practice, and the clearance times are 5 sec between incompatible signal groups. The maximum cycle time is set at 180 sec. The stopping criterion used in solution scheme is set when relative bound gap is less than 0.2%. Implementations for carrying out computations are made on DELL T7610, Intel Xeon 2.5 GHz processor with 32 GB RAM under Windows 10 using C++ compiler. To investigate the effectiveness and feasibility of FS, comparisons are also made with recently proposed NR and NR_W (Chiou 2016) and robust signal control (SR) by Yin (2008), as briefly summarized inTable 1. Results are summarized and plotted in Figs. 2-5. As seen in Fig. 2 for no-link closure HNDP, NR_W performs with the worst PI of all cases. Since NR considers no probabilistic uncertain risk, the PI stays constant as Γ grows. The proposed FS achieves a fairly low PI as Γ increases followed by a recently proposed SR. Moreover, let R and Z denote robust and nominal solutions in deterministic conditions as uncertainty budget $\Gamma = 0$. A relative percent loss of solution (SL) given by robust solution over that did in nominal situation can be described.

$$100 * \left(\frac{R - Z}{Z} \right) \% \quad \text{Eq. (33)}$$

As seen from Fig. 3, as expected, NR_W incurs a most high relative loss of all as Γ grows. By contrast, FS exhibits a relatively reliable better advantage over those did by recently proposed signal settings like SR. Considering a selected 6-link closure HNDP for varying uncertainty budget Γ in [0, 30] units, computational results can be summarized and thus plotted in Fig. 4. As seen in Fig. 4, the PI of all alternatives seems relatively higher than those in Fig. 2 for no-link closure to hazmat traffic. As expected, the NR_W taking no account of probabilistic uncertain risk on links incurs a most high PI in all cases. FS, again, exhibits a significant advantage over those did by SR as Γ grows, particularly. According to Eq. (33), a relative percent loss of robust solutions at deterministic conditions for 6-link closure HNDP can be summarized and thus plotted in Fig. 5. As expected, due to less adequate routes available to hazmat carriers in this scenario, the NR_W incurs a much higher relative optimality loss of all as compared to those in Fig. 3 for no-link closure HNDP. Again, FS achieves a most low sub-optimality of all as Γ grows.

Figure 1: Sioux Falls network with 6-link closure

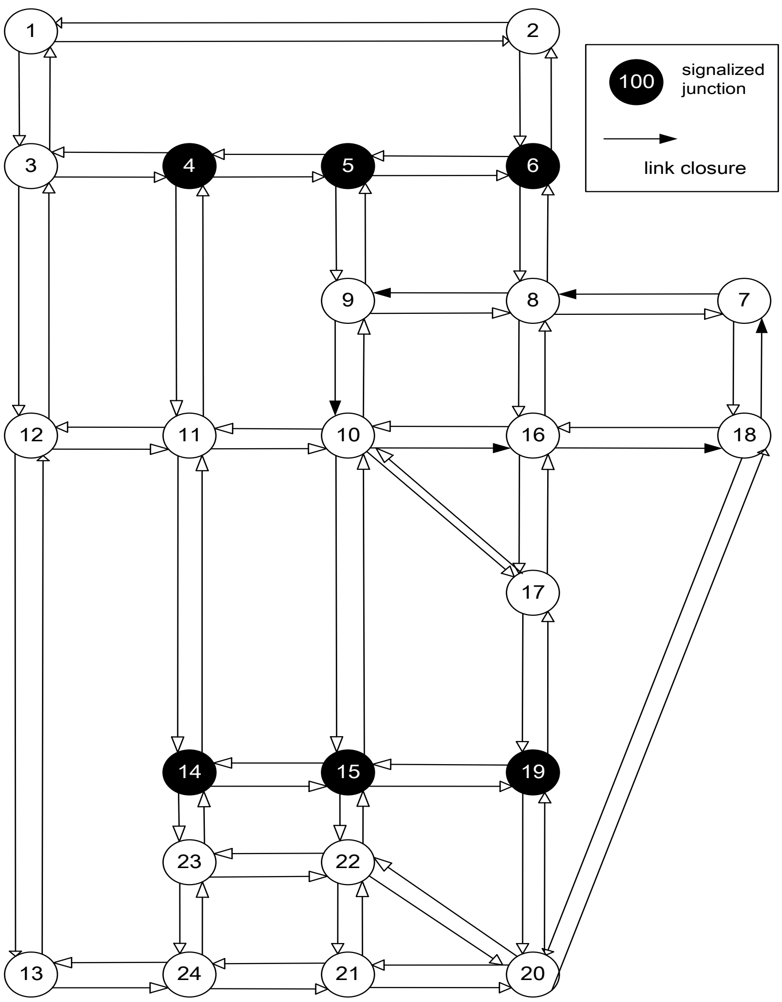


Table 1: Signal control strategies for urban road network under uncertainty

Signal settings	description	source
SR	Set-based robust signal control	Yin (2008)
NR	Deterministic signal control at nominal condition	Chiou (2016)
NR_W	Deterministic signal control at worst case	Chiou (2016)
FS	Flexible signal control	This paper

Figure 2: Computational results for no link closure at Sioux Falls network

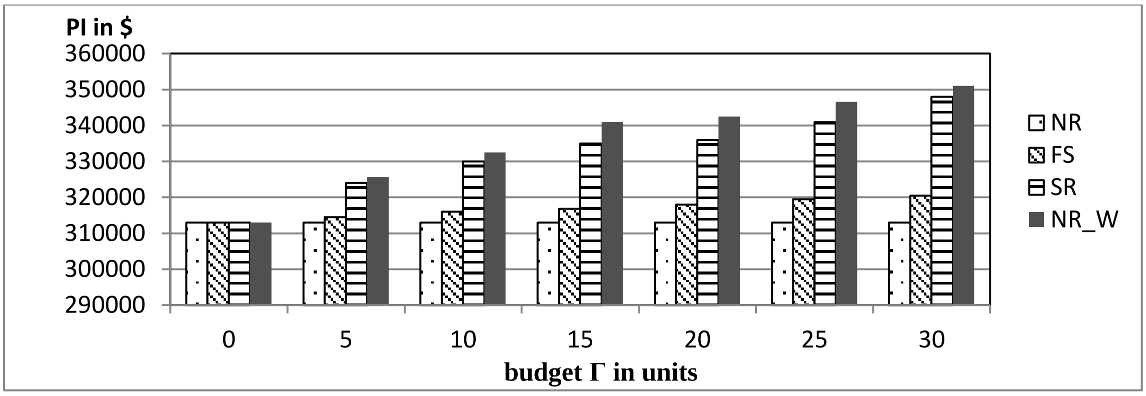


Figure 3: Optimality loss for no link closure at Sioux Falls network

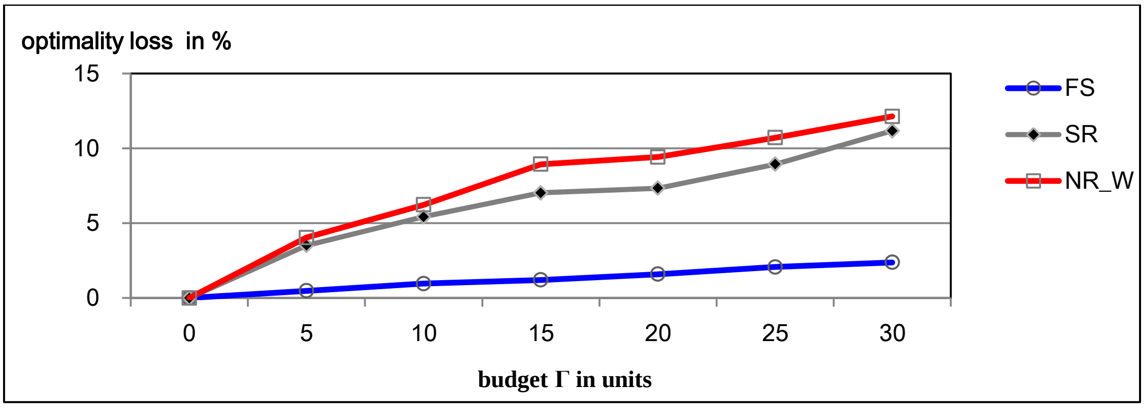


Figure 4: Computational results for 6-link closure at Sioux Falls network

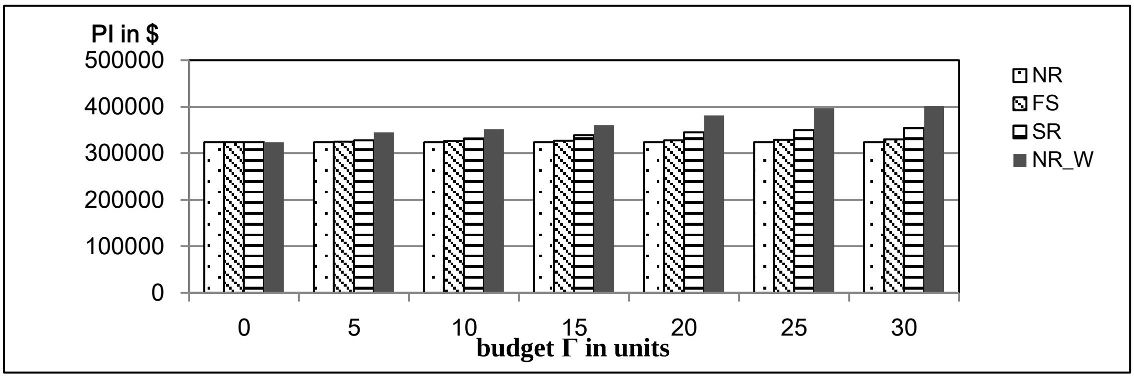
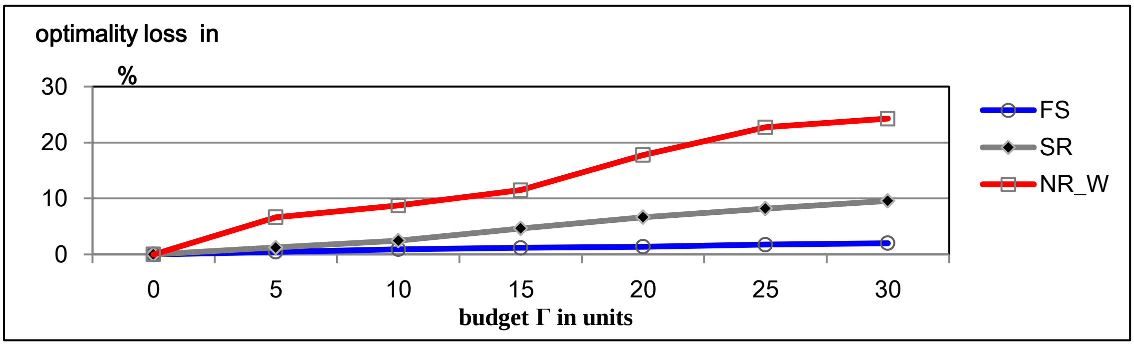


Figure 5: Optimality loss for 6-link closure at Sioux Falls network



7. Conclusion and discussion

In this paper, we considered a flexible signal setting (FS) against worst-case of uncertain risk. Following earlier results by the author, in this paper, we presented an effective scenario-based bi-level programming problem (SBLPP) for fixed-time signal settings. Due to non-convexity of the proposed model as widely known in the literature, it becomes notoriously difficult to solve the proposed model by current state-of-the-art methods with reasonable computational efforts. This paper presented a new solution method to solve proposed model with tractable computation. In order to demonstrate feasibility of proposed FS, numerical computation was performed using a medium-size real-data city network for a variety of uncertainty budgets. Computational comparison was conducted and made with recent well-known signal control like NR (Chiou 2016) and SR (Yin 2008) for no-link closure and 6-link closure. As it was shown, the results reported indicated the flexible robust signal settings can achieve relatively reliably better performance than those did. Considering practical issues about evaluation of time-varying transportation management, real-time implementations of signal control strategies need further investigation.

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